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Name: _____

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ISyE 2027 B
Test 3

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Let X be the demand for version 1 of a product, and Y be the demand for version 2 of the product. Let $D = X + Y$ be the total demand for the product. Assume that $\mathbb{P}\{X = i, Y = j\} = ij/15$ for integers i and j with $i \geqslant 1, j \geqslant 1$, and $i + j \leqslant 4$.

    (a) Compute $\mathbb{P}\{X = 2\}$.

    (b) Compute $\mathbb{P}\{X = 2 \mid Y = 2\}$.

    (c) Compute $\mathbb{E}[D \wedge 3]$.

2. (30 points) Let U be uniformly distributed over the interval $(0, 1)$, and let $X = U^2$.

    (a) Compute the kth moment of U where k is a non-negative integer.

    (b) What is the c.d.f. of X?

    (c) Find a median of X.

3. (30 points) Assume that Anna, Jathan, and Riley are all trying to solve a problem. Assume that the three times are independent, exponential random variables. Assume that the mean time for Jathan or Riley to solve the problem is 4 minutes or 1/15 of an hour, but the mean time for Anna is 5 minutes or 1/12 of an hour.

    (a) What is the expected time *in hours* until the first solution?

    (b) What is the probability that Anna solves the problem first?

    (c) What is the probability that Anna solves the problem last?

4. (30 points) Assume that Sam is an 80% free throw shooter; that is, on each free throw attempt, she is successful with probability 8/10. Suppose Sam starts shooting free throws.

    (a) What is the probability that the first miss is on the kth attempt for $k = 1, 2, \ldots$?

    (b) What is the expected number of attempts until the first miss?

    (c) Let S be the number of successful shots out of the first 100. What is the value of x so that $\mathbb{P}\{S \geqslant 85\} \approx \mathbb{P}\{Z \geqslant x\}$ where Z is a standard Gaussian (a.k.a. normal) random variable? Please use the continuity (a.k.a. histogram) correction if and only if it would be recommended for this situation.

5. (30 points) Suppose Z_1 and Z_2 are independent, standard Gaussian random variables. Let $X = 3Z_1 + 4Z_2$, and $Y = 4Z_1 + 3Z_2$.

    (a) Compute the mean of X.

    (b) Compute the covariance of X_1 and X_2.

    (c) Compute the standard deviation of X_1.