R. D. Foley
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Name: _____

ISyE 2027
Section B
Test 3

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose the c.d.f. of X is $F(t) = t^3/8$ for $0 \le t \le 2$.
   (a) Find the p.d.f. of X.
   (b) Find the mean of X.
   (c) Compute $\mathrm{E}[1/X^2]$.
2. (30 points) Suppose X is Poisson with mean 2, Y is Poisson with mean 3, and X and Y are independent. Compute the following.
   (a) $\mathrm{E}[X + Y]$.
   (b) $\mathrm{E}[X \wedge 1]$.
   (c) $\mathrm{Var}(X - Y)$.
3. (30 points) Service times at server 1 are exponentially distributed with mean 1/4 hour. Service times at server 2 are exponentially distributed with mean 1/3 hour. All service times are independent. Viviana is starting service at server 1. Denny is starting service at server 2. Ekta is waiting for the first available server.
   (a) What is the probability that Viviana leaves after Denny?
   (b) What is the expected time until the 2nd departure?
   (c) What is the probability that they depart in alphabetical order?
4. (30 points) Suppose Kendrick has 25 jobs to process. Each job is exponentially distributed with mean 1/5 hour. Assume that the processing times are independent. Let T be the total time to process the 25 jobs.
   (a) What is the mean of T?
   (b) What is the variance of T?
   (c) Accurately approximate the probability that Kendrick processes the jobs in 6 hours.
5. (30 points) Suppose (X, Y) is uniformly distributed over the triangle with corners at $(0, 0)$, $(0, 1)$, and $(1, 0)$. Let $T = X + Y$.
   (a) Compute $\mathrm{P}\{T \le z\}$ for $0 \le z \le 1$.
   (b) Find the marginal p.d.f. of X.
   (c) Are X and Y independent? Explain.