

ISyE 2027
Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers except for 2(c).

1. (30 points) On yesterday's ballot in DeKalb county, each voter needed to check either Republican, Democrat, or non-partisan. Consider the voters entering Briarlake Elementary School. What would be a reasonable guess for the distribution of each of the following?
 - (a) The number of Republican voters out of the first 100 voters.
 - (b) The number of voters until the first Democratic voter.
 - (c) The length of time from noon until the first non-partisan voter enters the building after noon.
2. (30 points) Let M be the number of heads among 100 tosses of a fair coin.
 - (a) What is the expected number of heads?
 - (b) What is the standard deviation of M ?
 - (c) The probability that M is exactly 50 is approximately what? (Come up with a number; do not leave terms with factorials, powers, etc.)
3. (30 points) Suppose you are waiting for a taxi, and the time until the next taxi is exponentially distributed with mean 5 minutes.
 - (a) What is the probability that you wait more than 5 minutes?
 - (b) What is the probability that your total waiting time is more than 15 minutes given that you have already waited 5 minutes?
 - (c) What is the variance of the waiting time including units?
4. (30 points) The length of time T to process a part has c.d.f. $F(t) = t^3/27$ for $0 \leq t \leq 3$.
 - (a) Determine the probability that the service time is less than 1.
 - (b) Compute the mean processing time.
 - (c) Compute $\mathbb{E} \left[\frac{1}{T^2} \right]$.
5. (30 points)
Let X and Y have joint p.m.f. given by $\mathbb{P}\{X = i, Y = j\} = (i + j)/9$ for $i = 0, 1, 2$ and $j = 0, 1$.
 - (a) Compute $\mathbb{E}[Y]$.
 - (b) Compute $\mathbb{E}[XY]$.
 - (c) Compute the squared coefficient of variation of Y . (If you do not recall the SCV, compute the variance of Y for half credit. Do only one of the two and indicate which one.)
6. (30 points) Suppose that a picker needs to walk down an aisle of length 40 feet to a random location L .
 - (a) If L were uniformly distributed over the length of the aisle, what would be the mean of L ?
 - (b) Suppose that 25% of the items cause 80% of the retrievals and that these most active items are placed in the first quarter of the aisle. What would be the mean of L ?
 - (c) Suppose that 25% of the items cause 80% of the retrievals and that these most active items are placed in the first quarter of the aisle. What would be the $\mathbb{P}\{L \leq 20\}$?