

ISyE 2027 B  
Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like  $\binom{52}{5}$  and  $e^{-2}$  in your answers.

1. (30 points) What would be a reasonable guess for the distribution of the following quantities?

- (a) The number of train derailments in the U.S. during December 2018.
- (b) The number of cars that enter an intersection until a car turns left.
- (c) The number of eggs that exceed 60 grams in a carton of a dozen eggs.

2. (30 points)

Suppose we roll a pair of fair dice—one red and one green. Let  $R$  be the event that the red die is a 2 and  $G$  be the event that the green die is a 2. Let  $A = R \cup G$ .

- (a) Compute  $\mathbb{P}(A)$ .
- (b) Compute the probability that both dice are twos given that there is at least one two.
- (c) Compute  $\mathbb{P}(R^c A)$ .

3. (30 points) Let  $X$  have p.d.f.  $f(t) = |t|$  for  $-1 < t < 1$ . Let  $F(t)$  be the c.d.f. of  $X$ .

- (a) Compute  $F(1/2)$ .
- (b) Compute  $\text{Var}(X)$ .
- (c) Compute  $\mathbb{P}\{X \leq 0 \mid X \leq 1/2\}$ .

4. (30 points) Tomorrow's demand  $D$  for a particular product has a Poisson distribution with mean 3. The product sells for \$10.

- (a) What is the probability that tomorrow's demand is 0?
- (b) What is the variance of  $10D$ ?
- (c) There are only 2 items in stock. Compute the expected number of items sold. (At most half credit if your answer has an infinite summation.)

5. (30 points) A picker is at the end of an aisle that is 40 meters in length. The picker needs to walk down the aisle to a random location  $L$ , pick the item, and return to the starting point. The picker walks at a speed of 2.5 meters per second and takes 4 seconds to pick the item. The high turnover items are stored in the first 10 meters of the aisle; the low turnover items are stored in the rest of the aisle. The high turnover items cause 90% of the activity. Assume that the p.d.f. of  $L$  is constant over the region  $(0, 10)$  and a possibly different constant over the region  $(10, 40)$ .

- (a) Let  $R$  denote the round trip travel time. Express  $R$  as a function of  $L$ , the walking speed, and the time needed to pick the item.
- (b) Compute  $\mathbb{P}\{L \leq 20\}$ .
- (c) Compute  $\mathbb{E}[L]$ .