

R. D. Foley
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Name: _____

ISyE 2027
Section C
Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose we are observing visitors to a web site. What is a reasonable guess as to the distribution of each of the following?
 - (a) The number of visitors until a visitor makes a purchase?
 - (b) The number of visitors during the next hour?
 - (c) The length of time until the next visitor arrives?
2. (30 points) Suppose the r.v. X has p.d.f. $f(s) = 4s - 3s^2$ for $0 \leq s \leq 1$.
 - (a) Compute the mean of X .
 - (b) Compute the c.d.f. of X .
 - (c) Compute $E[1/X]$.
3. (30 points) Suppose X has p.d.f. $f_X(s) = e^{-s}$ for $s \geq 0$, and $Y = X/5$.
 - (a) Compute $P\{X > 5 \mid X > 3\}$.
 - (b) What is the mean of Y ?
 - (c) Determine the p.d.f. of Y , $f_Y(s)$.
4. (30 points) Suppose Dido instead used the rope of length L to create a rectangle along the coastline. Assume that the side parallel to the coast has a length $L/2$, and the two sides that are orthogonal to the coastline each have length $L/4$. Assume that L has mean 2 and standard deviation σ .
 - (a) What is the expected area A of the rectangle as a function of σ ?
 - (b) What is the support of the area A assuming that L is uniformly distributed from 0 to 4?
 - (c) What is the c.d.f. $F_A(t)$ of A assuming that L is uniformly distributed from 0 to 4?
5. (30 points) Suppose 100 people will be requesting meals. Each person selects a vegetarian meal with probability $1/5$, and their selection is independent of the selection of the other people. Let N denote the number of people who select a vegetarian meal.
 - (a) What is the mean of N ?
 - (b) What is the variance of N ?
 - (c) Accurately approximate the probability that 28 vegetarian meals will be sufficient to satisfy demand.