R. D. Foley
November 7, 2022

Name: _____

ISyE 2027
Section B
Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and $e^{-2}$ in your answers.

1. (30 points) Suppose the p.d.f. of $X$ is $f(t) = t^3/4$ for $0 \le t \le 2$.
   (a) Find the mean of $X$
   (b) Find the c.d.f. of $X$.
   (c) Compute $\mathrm{E}[4/X]$.
2. (30 points) What is a reasonable guess as to the distribution of each of the following?
   (a) The length of time until I hear a plane flying overhead.
   (b) The number of planes that fly overhead during the next 60 minutes.
   (c) The number of Cessnas out of the next 10 planes that fly overhead.
3. (30 points) Suppose we have 3 items in stock. The demand $D = X - 1$ where $X$ has a geometric distribution with mean 4. Let $N$ be the number sold.
   (a) Express $N$ as a function of $D$.
   (b) Compute $\mathrm{P}\{N = 0\}$.
   (c) Compute $\mathrm{P}\{N = 3\}$.
4. (30 points) The number of people $N$ that will attend an upcoming event has a mean of 200 and a variance of 400. The number of hors d'oeuvres needed is $X = 3(N + 20)$.
   (a) What is the mean of $X$?
   (b) What is the variance of $X$?
   (c) Can you compute a good upper bound $\mathrm{P}\{|N - 200| \ge 60\}$?
5. (30 points) Suppose drivers must turn either left or right, and they have an equal probability of turning left or right. Assume that one driver's decision is not affected by the decisions of the other drivers. Let $N$ be the number of drivers that turn right out of the next 81 cars.
   (a) What is the mean of $N$?
   (b) What is the variance of $N$?
   (c) Accurately approximate $\mathrm{P}\{N \le 49\}$. Use the continuity correction if appropriate.