

ISyE 2027  
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like  $\binom{52}{5}$  and  $e^{-2}$  in your answers.

1. (30 points)

About 90% of shishito peppers are mild, but the other 10% can be spicy. The only way to know is to taste it.

- (a) What would be a reasonable guess for the distribution of the numbers of shishito peppers eaten until eating a spicy pepper? Include the values of any parameters.
- (b) What would be a reasonable guess for the distribution of the number of spicy peppers among the first dozen eaten? Include the values of any parameters.
- (c) The number of spicy peppers among the first 4 dozen eaten would be approximately distributed as what distribution that has a single parameter? Include the value of the parameter. (In other words, do not specify a distribution that involves two or more parameters.)

2. (30 points) A company has 3 suppliers for a particular item. Supplier A supplies 70%, and Supplier B supplies 20%. One percent of Supplier A's items are defective. Both Supplier B and C's items are defective with probability .02.

- (a) Given that an item did not come from C, what is the probability that it came from A?
- (b) What proportion of the received items are defective?
- (c) Given that an item is defective, what is the probability that the item came from A?

3. (30 points) Assume that you are dealt seven cards from a well-shuffled standard deck.

- (a) What is the probability of being dealt 2 triples (but not 4 of a kind)?
- (b) What is the probability of being dealt three of one kind and four of another?
- (c) What is the probability of 7 hearts given that you have at least 6 hearts? (Please work this one out completely for full credit; that is, do not leave your answer in terms of  $\binom{n}{k}$ .)

4. (30 points) Suppose that the daily demand  $D$  for a particular item is such that  $\mathbb{P}\{D = 1\} = 1/10$ ,  $\mathbb{P}\{D = 2\} = 7/10$ , and  $\mathbb{P}\{D = 3\} = 2/10$ . Tomorrow morning, we will only have 2 items in stock when opening for business.

- (a) Compute tomorrow's expected demand.
- (b) Compute tomorrow's expected number of items sold.
- (c) Compute the second moment of  $D$ .

5. (30 points) Suppose the demand  $N$  is non-negative with mean 15 and that  $Y = 3N + 7$ .

- (a) What is the best upper bound that you can obtain for  $\mathbb{P}\{N \geq 100\}$ ?
- (b) Compute  $\mathbb{E}[Y]$ .
- (c) If  $\sigma^2$  denotes the variance of  $N$ , what is the variance of  $Y$ ?

6. (30 points) Suppose that  $\mathbb{P}\{X = k\} = ke^{-\lambda}\lambda^{k-1}/k!$  for  $k = 0, 1, 2, \dots$  where  $\lambda > 0$  (which we might call a "not-quite-a-Poisson" distribution).

- (a) Compute  $\mathbb{P}\{X = 1 \mid X \leq 2\}$ .
- (b) Compute the generating function  $G(z) = \mathbb{E}[z^X]$ .
- (c) Compute the mean of  $X$ .