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Name: _____

ISyE 2027 B
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose we roll a pair of fair dice—one red and one green. Let R be the event that the red die is a 6 and G be the event that the green die is a 6. Let $A = R \cup G$ and $B = RG$.

   (a) Compute $\mathbb{P}(A)$.

   (b) Compute $\mathbb{P}(R \mid A)$.

   (c) Compute $\mathbb{P}(R^c A)$.

2. (30 points)

   There are 3 coins. They look identical, but two are fair, and the other is unfair. The unfair coin comes up heads with probability 3/4. You select one of the coins at random, and flip it twice.

   (a) What is the probability that you select a fair coin and get tails on both coin flips?

   (b) What is the probability that both coin flips come up heads?

   (c) Given that both coin flips came up heads, what is the probability that you were flipping the unfair coin?

3. (30 points) Assume that you are dealt four cards from a well-shuffled standard deck.

   (a) What is the probability of being dealt two pairs?

   (b) What is the probability of being dealt three of a kind (i.e., three of one rank)?

   (c) What is the probability of 4 aces given that you have at least 3 aces? (Please work this one out completely for full credit; that is, do not leave your answer in terms of $\binom{n}{k}$.)

4. (30 points) Suppose that the random variable X has mean 3 and standard deviation 4. Let $Y = 5X + 6$.

   (a) The squared coefficient of variation of X is defined to be the $\mathrm{Var}[X]/(\mathbb{E}[X]^2)$. What is the squared coefficient of variation of X in this problem?

   (b) Compute the mean of Y.

   (c) Compute the standard deviation of Y.

5. (30 points) Suppose the demand D for a particular product has the following probability mass function:

$$\mathbb{P}\{D = k\} = \begin{cases} (4-k)/10 & \text{for } k = 0, 1, 2, 3 \\ 0 & \text{otherwise.} \end{cases}$$

   However, there are only 2 items in stock, and no more items are available. Let N be the number of items sold.

   (a) What is the expected demand?

   (b) What is the second moment of D?

   (c) Compute $\mathbb{E}[N]$.