R. D. Foley   Name: _____
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ISyE 2027
Section C
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose $P(A) = 3/10$, $P(B) = 4/10$ and $P(B \mid A) = 2/3$. Compute the following:

   (a) $P(A^c)$,

   (b) $P(A \cup B)$,

   (c) $E\left[\mathbb{1}_{\{AB\}} - \mathbb{1}_{\{A\}}\mathbb{1}_{\{B\}}\right]$.

2. (30 points) Suppose we take a standard deck of 52 cards and remove all 13 hearts leaving a deck of 39 cards. Assume we are dealt 5 cards.

   (a) What is the probability of a full house, i.e., 3 cards of one rank and 2 cards of another?

   (b) What is the probability of two pairs (and not 3 of any rank)?

   (c) What is the probability of getting the king of diamonds, one other king, and all 3 queens?

3. (30 points) Suppose we roll a red die and a green die. Let X be the value on the red die and Y the value on the green die.

   (a) Compute the probability that $XY = 30$.

   (b) Compute the mean of $X \wedge 2$.

   (c) What is the probability that both dice are less than or equal to 2 given that there is at least one die less than or equal to 2?

4. (30 points) Suppose X has p.m.f. $P\{X = k\} = (k + 1)/6$ for $k = 0, 1, 2$. Compute

   (a) the mean of X,

   (b) the second moment of X,

   (c) $E[1/(X + 1)]$.

5. (30 points) Suppose that there are 2 items in stock in the morning. The demand D during the day has p.m.f. $P\{D = k\} = (k + 1)/10$ for $k = 0, 1, 2, 3$. There will be no restocking during the day.

   (a) What is the probability that both items will be sold that day?

   (b) What is the expected number sold that day?

   (c) Let R be the remaining stock at the end of the day. Give an expression defining R as a function of D.