R. D. Foley
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Name: _____

ISyE 2027
Section B
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose $\mathrm{P}(A) = 2/10$, $\mathrm{P}(B) = 3/10$, and $\mathrm{P}(A \mid B) = 1/3$. Compute the following.
   (a) $\mathrm{P}(A^c)$.
   (b) $\mathrm{P}(A \cup B)$.
   (c) Are B and the empty set *both* mutually exclusive and independent?

2. (30 points) Suppose we take a standard deck and remove the 4 aces leaving 48 cards with ranks: 2, 3, 4, . . . , 10, J, Q, K. Assume that we're dealt 7 cards.
   (a) What is the probability that we have 4 of one kind and 3 of another?
   (b) What is the probability that we have 3 pairs (and not 3 of any kind)?
   (c) What is the probability of a straight with only hearts? (A straight here means 7 cards in a row, e.g., having a 4, 5, 6, 7, 8, 9, and 10.)

3. (30 points) Suppose X has p.m.f. $\mathrm{P}\{X = k\} = k/6$ for $k = 1, 2, 3$. Compute the following:
   (a) the mean of X,
   (b) the second moment of X,
   (c) $\mathrm{E}[1/X]$.

4. (30 points) Suppose we roll a red die and a green die. Let X be the value on the red die and Y the value on the green die.
   (a) Compute the probability that $Y/X = 2$.
   (b) Compute the mean of $(X - 5)^+$.
   (c) What is the probability that both dice are greater than or equal to 5 given that at least one die is greater than or equal to than 5?

5. (30 points)

   Suppose we will receive 1000 dollars at a random time N periods from now. To make the calculations simpler, assume that the effective interest rate per period is 50%. Let Y denote the present value of the 1000 dollars. The random variable N has probability generating function $G(z) = 0.7 + 0.3z$.
   (a) Compute the mean of N.
   (b) Give an expression for Y as a function of N.
   (c) Compute $\mathrm{E}[Y]$.