ISyE 2027 Probability with Applications Fall 2002
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Homework 8

November 7, 2002

Due on Tuesday, Nov. 12

1. Suppose a beagle is working in customs inspecting passengers and luggage for banned substances, and the beagle alerts the handler by sitting next to the location of the banned substance. What would be a reasonable guess for the distribution of each the following: (a) the number of alerts by the beagle during the next 3 hours, (b) the length of time until the next alert, (c) the number of bags sniffed before alerting, (d) whether or not the beagle alerts when sniffing the next bag, (e) the number of alerts out of the next 25 bags sniffed, and (f) the combined weight of the next 25 bags that the beagle indicates are carrying banned substances?

2. Suppose X and Y are random variables with joint probability density function $f_{X,Y}(s,t) = ct$, for positive s and t with $s + t \leq 1$. a) determine c, b) determine the marginal p.d.f. of X, c) determine the marginal p.d.f. of Y, d) the mean and variance of X, e) the mean and variance of Y, f) the conditional p.d.f. of Y given $X = 1/2$, g) the covariance of X and Y, h) $\Pr\{X = Y\}$ i) $\mathrm{E}[2X + 3Y]$ and j) $\mathrm{Var}[Y + X]$.

3. Suppose we have an AS/RS with a square storage rack that is 20 meters long. The retrieval device has two motors: one that moves the mast up and down the aisle, and the other raises and lowers device which removes the tray from the rack. Assume that the lower left corner of the rack is designated (0,0) and that a tray must be retrieved from a random location (X, Y). Assume that 25% of the items cause 80% of the activity, and that these active items have been stored in the lower left corner of the rack.
   (a) Find the joint probability density function of (X, Y).
   (b) Assuming that each motor causes the retrieval device to move at 2 meters per second either horizontally or vertically, and that the travel time is the maximum of the horizonal travel time and the vertical travel time. Let T be the travel time from the origin to (X, Y). Find the cumulative distribution function of T.
   (c) Are X and Y independent? Explain.

4. Suppose a worker needs to process 100 items. The time to process each item is exponentially distributed with a mean of 3 minutes, and the processing times are independent. What is the probability that the worker finishes in less than 6.25 hours?