ISyE 2027 Probability with Applications Fall 2003
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Homework 6
November 28, 2004
due on Wednesday

1. Suppose Tucker the beagle is working in customs inspecting passengers and luggage for banned substances, and Tucker alerts the handler by sitting next to the location of the banned substance. What would be a reasonable guess for the distribution of each the following: (a) the number of alerts by Tucker during the next 3 hours, (b) the length of time until the next alert, (c) the number of bags sniffed before alerting, (d) whether or not Tucker alerts when sniffing the next bag, (e) the number of alerts out of the next 25 bags sniffed, and (f) the combined weight of the next 25 bags that Tucker indicates are carrying banned substances?

2. Suppose X has p.m.f. $\Pr\{X = k\} = c(k + 2)$ for $k = -1, 0, 1, 2$. Find c, and compute the mean, variance, and standard deviation of X. Let $Y = 3X + 5$. Compute the mean, variance, and standard deviation of Y.

3. Suppose X has pdf $f(s) = c(1+s)$ for $-1 \le s \le 1$. Determine c and the mean, variance, and standard deviation of X. Let $Y = 3X + 5$. Compute the mean, variance, and standard deviation of Y.

4. Let X and Y have joint probability density function $f_{(X,Y)}(s,t) = ce^{-(s+2t)}$ for $0 \le s$, and $0 \le t$. Find (a) c, (b) $\Pr\{\min(X, Y) > 1/3\}$, (c) $\Pr\{X \le Y\}$, (d) the marginal probability density function of X, and (e) $\mathrm{E}[XY]$.

5. Let X and Y be independent uniform $(0, 1)$ random variables. Compute (a) $\Pr\{X < Y\}$, (b) $\Pr\{X = Y\}$, (c) the probability density function of $X + Y$, (d) $\mathrm{Var}[X]$, and (e) $\mathrm{Var}[X + Y]$.

6. Suppose we have a building with a floor shaped like an isosceles right triangle. The two sides adjacent to the right triangle have length 100 feet. Think of the right angle being at the origin, and other two corners at $(100, 0)$ and $(0, 100)$. The overhead crane is located at the origin and needs to travel to a point (X, Y) which is uniformly distributed over the region. The crane has two motors one that moves the crane north and south, and the other that moves the crane east and west. Both motors move at the speed of 20 feet per minute. Since the motors can work at the same time, it is reasonable to assume that the length of time to go from the origin to (X, Y) is the maximum of two times: the time to go east from 0 to X, and the time to go north from 0 to Y. Let T be the length of time that it takes the crane to move from the origin to (X, Y) and return to the origin. [40]

   (a) What is the joint probability density function of (X, Y)?
   (b) Find the marginal probability density function of X.
   (c) Use symmetry to determine the marginal pdf of Y without integrating.
   (d) What is the conditional p.d.f. of Y given $X = 10$?
   (e) What is the $\mathrm{E}[Y \mid X = 10]$?
   (f) Are X and Y independent? Explain.
   (g) Derive an expression for the round trip time T as a function of X, Y, and the speed of the motors.
   (h) Find $\Pr\{T > 2.5 \text{ minutes}\}$.
   (i) Compute $\mathrm{E}[T]$
   (j) Do you think X and Y are positively correlated, negatively correlated, or uncorrelated? Why?

7. Suppose the average weight of an item labelled 16 ounces actually has mean 17 ounces and variance 4 ounce^2. What is the approximate probability that the combined weight of 25 items exceeds 445 ounces? What theorem is useful in answering this question?