Homework 11

Due: never, but simlar problems will most likely appear on the final

1. Suppose a worker needs to pick 100 items. The mean time to pick each item is 5 minutes with variance with variance 9 minutes^2. Assuming that the pick times are independent and identically distributed, what is the approximate probability that the total time to pick the 100 items is less than 550 minutes?

2. Suppose the number of items in an order has a binomial distribution with $n = 15$ and $p = 1/5$. Compute the probability of 4 items in the order. Using the central limit theorem, approximate the probability of 4 items in the order. What is the difference between the two probabilities. Also, use the clt to compute the probability of 4 or more orders and also strictly more than 4 orders.

3. Suppose you are selling lemonade at a game, and you cannot get more lemonade during the game if you sell out. The lemonade retails for \$15 per gallon, but only costs you \$1 per gallon. Leftover lemonade can be sold at 10 cents per gallon. If the demand is exponentially distributed with mean \$500 gallons, how many gallons should you have on hand before the game. If the demand were uniformly distributed between 300 and 700 gallons, how many gallons should you have on hand at the beginning of the game?

4. item Suppose we have a miniload with a square rack of length 100 with the i/o point at the lower left corner. Both motors move at the speed of 20 feet per minute. Since the motors can work at the same time, it is reasonable to assume that the length of time to go from the origin to (X, Y) is the maximum of two times: the time to go east from 0 to X, and the time to go north from 0 to Y. Let T be the length of time that it takes the crane to move from the origin to (X, Y). Assume that the items stored in the rack have been divided into two groups: high turnover and low turnover. The 25% most active are responsible for 90% of the activity, and are stored randomly in the lower left hand quadrant. The remaining 75% are stored randomly in the remaining L-shaped region of the rack.

   (a) Determine the joint probability density function of (X, Y)?

   (b) Find the marginal probability density function of X.

   (c) Use symmetry to determine the marginal pdf of Y without integrating.

   (d) Are X and Y independent? Explain.

   (e) Derive an expression for the round trip time T as a function of X, Y, and the speed of the motors.

   (f) Find $\Pr\{T > 2.5 \text{ minutes}\}$.

   (g) Find the conditional probability density function of X given $Y =$

   (h) Write down the double integral for $E[T]$ but do not integrate it.

   (i) Do you think X and Y are positively correlated, negatively correlated, or uncorrelated? Why?