Name: _____

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ISyE 2027c

Test 3

Calculators, notes, and books are not allowed. Please work in the bluebook and transfer your answers to the back of this sheet. Put your name on everything and hand in both the bluebook, test, and answer sheet.

1. (30 points) What would be the name of a reasonable distribution for modelling the following? (a) The number of eggs weighed until an egg exceeds 65 grams. (b) The number of eggs out of the next 100 that exceed 65 grams. (c) Whether or not the combined weight of the next 100 eggs exceeds 6.5 kilograms. (d) The number of specks of pepper in one cubic inch of an omelet. (e) The combined volume of 100 yolks plus whites.

2. (30 points) Suppose X is equally likely to be 0 or 2. (a) What is the mean of X? (b) What is the variance of X? (c) What is $\Pr\{X = 2 \mid X \neq 2\}$?

3. (30 points) Let X be the number of hearts and Y the number of clubs in a 5 card hand dealt from a standard deck. Find the marginal probability mass function of X. Find the joint probability mass function of X and Y. (In answering the previous two questions, please use the notation $\binom{n}{k}$ and leave your answer as an equation in terms of i and j; do not make a table of all the cases on your answer sheet.) Are X and Y independent and why?

4. (30 points) Suppose X has probability density function $f(s) = |s|$ for $-1 \leq s \leq 1$. (a) Find the mean of X. (b) Find the variance of X. (c) Find the cumulative distribution function of X.

5. Suppose the average amount of juice in a randomly selected orange is 1/3 cup with a standard deviation of 1/3 cup, and that we need 21 cups of juice. Accurately approximate the probability that 81 oranges will supply enough juice.

6. Suppose a street vendor sells fresh squeezed orange juice at a price of \$10 per gallon though it only costs the vendor \$6 per gallon. The vendor is unable to replenish the juice supply during the day. Leftover juice has a value of \$1 per gallon. Assume that the probability density function of the daily demand for juice is $f(s) = [(s - 10)/30]^2$ for $10 \leq s \leq 40$ gallons. How many gallons of juice should the vendor bring each morning in order to maximize the expected profit?

7. (30 points) Suppose an overhead crane operates in a building that is 90 feet by 90 feet. Let us label the center of the building as (0,0), and the four corners are at $(\pm 45, \pm 45)$. One motor moves the crane north and south at 5 feet per second; the other motor moves the crane east and west at 5 feet per second. Let T be time for the the crane to go from the center of the building to a random point (X, Y) that is uniformly distributed over the floor of the building. Find the cumulative distribution function of T.

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