Name: _____

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ISyE 2027
Test 3

Calculators, notes, and books are not allowed. Please your answers to the back of this sheet. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose 5 marbles are drawn at random from an urn containing 9 red, 9 blue, 9 green, and 9 yellow marbles. (a) What is the probability that the 5 marbles have 3 of one color and 2 of another color? (b) What is the probability that the 5 marbles have 2 of one color, 2 of another color, and 1 of a third color? (c) What is $\Pr\{X = k\}$ where X is the number of red marbles among the five?

2. (30 points) Suppose that Melissa models the income level Y as having a Pareto distribution with parameter $\alpha = 1$. Compute (a) the p.d.f. of Y, (b) the mean of Y, and (c) the median of Y. (The c.d.f. of a Pareto distribution with parameter $\alpha > 0$ is $F(s) = 1 - 1/s^\alpha$ for $s > 1$.)

3. (30 points) Suppose X has mean 2 and variance 16. Let $Y = -3X + 1$. (a) What is the variance of Y? (b) What is $\mathrm{E}[Y \mid X = 1]$? (c) What is $\mathrm{E}[XY]$?

4. (30 points) Suppose X has a geometric distribution with parameter p. Suppose the conditional distribution of Y given $X = i$ is Poisson with mean i. Compute (a) $\Pr\{X = 2, Y = 0\}$, (b) $\mathrm{E}[Y \mid X = 2]$, and (c) $\Pr\{Y = 0\}$.

5. (30 points) Let $S = X_1 + \cdots + X_{36}$ where $X_1, \ldots, X_{36}$ are i.i.d., continuous random variables with mean 10 and variance 25. (a) What is the mean of S. (b) What is the standard deviation of S? (c) Approximate $\Pr\{S > 420\}$.

6. (30 points) Suppose (X, Y) is uniformly distributed over a triangle with corners at (0,0), (1,0), and (0,1). Compute (a) the marginal p.d.f. of X, (b) $\mathrm{E}[XY]$, and (c) $\Pr\{Z \leq 1/3\}$ where $Z = X + Y$.