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Name: _____

ISyE 2027
Test 3

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose you are watching traffic from the 14th street bridge. What would be a reasonable guess for the distribution of the following random variables?
   (a) The number of cars passing by until the first convertible.
   (b) The number of convertibles that pass by in the next hour.
   (c) The length of time until the first convertible.

2. (30 points) The four aces have been removed from a standard deck of cards leaving 48 cards. Suppose Emma is dealt 5 cards.
   (a) What is the probability of Emma being dealt a full house?
   (b) What is the probability of Emma being dealt two pairs?
   (c) What is the probability of Emma being dealt 5 hearts?

3. (30 points + bonus) Suppose X and Y have joint p.m.f.
$$\mathbb{P}\{X = i, Y = j\} = \begin{cases} 1/36 \text{ for } 1 \leqslant i = j \leqslant 6 \\ 2/36 \text{ for } 1 \leqslant i < j \leqslant 6. \end{cases}$$
   (a) What is $\mathbb{P}\{Y - X = 4\}$?
   (b) What is $\mathbb{P}\{X = 1 \mid Y = 3\}$?
   (c) What is $\mathbb{E}[X \mid Y = 3]$?
   (d) Bonus: Can you describe a real world example which would have this joint p.m.f.?

4. (30 points) Suppose that X has a p.d.f. $f_X(s) = 3s^2$ for $0 < s < 1$ and is zero otherwise. (a) Compute $\mathbb{E}[X]$. (b) Compute the c.d.f. of X. (c) Suppose that the random variable U has a uniform $(0, 1)$ distribution. How should you define $g(U)$ so that $g(U)$ has the same distribution as X?

5. (30 points) Suppose $Z = 3X - Y + 4$ where X has mean 10 and variance 9, Y has mean 5 and variance 2, and X and Y are independent.
   (a) What is the expected value of Z?
   (b) What is the variance of Z?
   (c) What is the covariance of Y and Z?

6. (30 points) Suppose that Patrick has 3 dozen eggs. Assume that the weights of the eggs are i.i.d random variables with mean 60 grams and variance 9 grams^2. Let W denote the total weight of the 36 eggs?
   (a) What is the standard deviation of W (including units)?
   (b) Approximately, what is $\mathbb{P}\{W > 2178\}$?
   (c) What would be a tight upper bound for $\mathbb{P}\{|W - 2160| > 9\}$?