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Name: ______________________________

ISyE 2027
Test 3

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) A bowl contains 100 jelly beans: 25 licorice, 25 orange, 25 lemon and 25 lime. Tom reaches in and removes 10 jelly beans at random.
   (a) What is the probability that all ten are licorice?
   (b) What is the probability of 5 one flavor and 5 of another?
   (c) What is the probability of 6 of one flavor and 4 of another?

2. (30 points) Suppose X and Y have joint p.d.f.

$$f_{(X,Y)}(s,t) = 2, \text{ for } 0 < t < s < 1, \text{ and } 0 \text{ otherwise.}$$

   (a) What is $\mathbb{E}[X]$?
   (b) What is $\mathbb{P}\{Y < X^2\}$?
   (c) What is the conditional p.d.f. of Y given $X = 3/4$?

3. (30 points) Rachel rolls a die until a 6 appears. Let N be the number of rolls. (a) What is the p.m.f. of X? (b) What is $\mathbb{P}\{X > 5 \mid X > 3\}$? (c) What is $\mathbb{E}[z^N]$?

4. (30 points) Suppose $Z = 3X - 4Y + 7$ where X has mean 5 and variance 9, Y has mean 2 and variance 2, and X and Y are independent.
   (a) What is the expected value of Z?
   (b) What is the variance of Z?
   (c) What is the covariance of X and Z?

5. (30 points) Suppose that Anna must process 25 jobs. Assume that the processing times of the 25 jobs are independent, exponentially distributed random variables with mean 4 minutes. Let T denote the total time in minutes to process the 25 jobs.
   (a) What is the mean of T (including units)?
   (b) What is the standard deviation of T (including units)?
   (c) Approximately, what is $\mathbb{P}\{T > 80\}$?

6. (30 points) Suppose that X has a Poisson distribution with mean 4. Define $p_k = \mathbb{P}\{X = k\}$ and $r_{k+1} = p_{k+1}/p_k$ for $k = 0, 1, \ldots$
   (a) What is the standard deviation of X?
   (b) Give a simple expression for r_{k+1}.
   (c) We derived Klar's upper bound on $\mathbb{P}\{X \geqslant m\}$ assuming that X had a binomial distribution. The same approach can be used for other distributions including when X is Poisson. The start of the derivation is

$$\begin{aligned} \mathbb{P}\{X \geqslant m\} &= \mathbb{P}\{X = m\} + \mathbb{P}\{X = m+1\} + \mathbb{P}\{X = m+2\} + \ldots \\ &= \mathbb{P}\{X = m\}(1 + r_{m+1} + r_{m+1}r_{m+2} + \ldots) \\ &\leqslant \ldots \end{aligned}$$

   What upper bound does Klar's approach give for $\mathbb{P}\{X \geqslant 12\}$ assuming X is Poisson with mean 4?