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## ISyE 2027D
## Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and $e^{-2}$ in your answers.

1. (30 points) Suppose traffic is light, and the speeds of cars going by a police car stopped on the side of the road are i.i.d. random variables.

   (a) What would be a reasonable guess for the distribution of the number of cars that go by in the next hour?

   (b) What would be a reasonable guess for the distribution of the number of cars that go by until a driver is given a speeding ticket?

   (c) What would be a reasonable guess for the distribution of the number of red cars out of the next 15 cars?

2. (30 points) Suppose that a random variable $X$ has c.d.f. given by $F(s) = s^2/4$ for $0 \leqslant s \leqslant 2$.

   (a) Compute the mean of $X$,

   (b) Compute the median of $X$,

   (c) Compute $\mathbb{P}\{1/3 < X \leqslant 1/2\}$,

   (d) Compute $\mathbb{P}\{X = 1/2\}$,

   (e) Compute the mean of $1/X$,

   (f) How could you generate $X$ from a uniform $(0, 1)$ random variable $U$?

3. (30 points) Suppose that $(X, Y)$ has a joint p.m.f. $\mathbb{P}\{X = i, Y = j\} = ij/15$ for integers $i$ and $j$ with $i \geqslant 1$, $j \geqslant 1$, and $i + j \leqslant 4$; otherwise, the probability is zero.

   (a) Compute the marginal probability $\mathbb{P}\{X = 2\}$.

   (b) Compute the conditional probability $\mathbb{P}\{X = 2 \mid Y = 1\}$.

   (c) Compute $\mathbb{E}[X \wedge Y]$.

4. The covariance of $X_i$ and $X_j$ is given by the $i, j$th element in the following matrix where $i$ and $j$ can be 1 or 2. For example, the covariance of $X_1$ and $X_1$ is given by the upper left element of the following matrix.
$$\begin{bmatrix} 9 & 5 \\ 5 & 16 \end{bmatrix}$$

   (a) What is the $\mathrm{Cov}(X_2, X_1 + X_2)$?

   (b) What is the $\mathrm{Cov}(8X_1, 5X_2)$?

   (c) What is the variance of $X_1 + X_2$?

5. (30 points) Assume that a picker needs to pick 100 orders. The processing times for the orders are i.i.d. with mean 2.3 minutes and variance $1/4$ minute$^2$.

   (a) Accurately approximate the probability that the picker finishes picking all 100 orders within 4 hours.

   (b) Accurately approximate the probability that the picker finishes picking all 100 orders within 237.5 minutes.

   (c) Determine a (useful) upper bound on the probability that the total time to pick all 100 hours is outside of the interval $(200, 260)$.