

ISyE 2027D  
Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like  $\binom{52}{5}$  and  $e^{-2}$  in your answers.

1. (30 points) Suppose that  $\mathbb{P}(A) = 6/10$ ,  $\mathbb{P}(B) = 5/10$ , and  $\mathbb{P}(A \cup B) = 9/10$ .
  - (a) Compute  $\mathbb{P}(A^c)$ .
  - (b) Compute  $\mathbb{P}(A | B)$ .
  - (c) Compute  $\mathbb{P}(A | B^c)$ .
2. (30 points) Suppose that  $\mathbb{P}\{X = 2\} = 3/6$ ,  $\mathbb{P}\{X = 5\} = 2/6$ , and  $\mathbb{P}\{X = 10\} = 1/6$ .
  - (a) Compute the mean of  $X$ .
  - (b) Compute the largest median of the distribution of  $X$ .
  - (c) Compute the mean of  $\sqrt{X-1}$ .
3. (30 points) A particular part is supplied by two different suppliers: A and B. Suppose we have on hand 7 parts from A and 3 parts from B. Each part from A is defective with probability one percent, while each part from B is defective with probability 2%. Assume that the parts are independent.
  - (a) What is the probability that a part randomly selected from the 10 parts is defective?
  - (b) What is the variance of the number of defective parts among the 10 parts?
  - (c) A part is selected at random and is determined to be defective. What is the probability that it was produced by A?
4. (30 points) Suppose that a miniload AS/RS has a storage rack that is 20 meters long and 10 meters high. The retrieval device has two motors: one that moves the device horizontally at 4 meters per second and another that moves the device vertically at 2 meters per second. Since the motors can operate simultaneously, the time that it takes to travel from the origin to a location  $(X, Y)$  is the maximum of the time to move horizontally and the time to move vertically. Assume that  $(X, Y)$  has a uniform distribution over the rack face.
  - (a) Let  $T$  be the travel time in seconds from the origin to  $(X, Y)$ . Write down an expression for  $T$  as a function of  $(X, Y)$  and the speeds of the motors.
  - (b) Compute  $\mathbb{P}\{T \leq 4 \text{ seconds}\}$ .
  - (c) Compute  $\mathbb{P}\{X > Y\}$ .
5. (30 points) Assume that a picker needs to pick 25 orders. The times to pick the orders are i.i.d. with mean 4 minute and variance 4 minutes<sup>2</sup>. Let  $T$  be the total time to pick all 25 orders.
  - (a) Accurately approximate the probability that  $T$  is less than 90 minutes.
  - (b) How much time would the picker need to be 90% sure of finishing all of the orders?
  - (c) Can you come up with a (useful) lower bound on  $\mathbb{P}\{70 < T < 130\}$ ; that is,  $\mathbb{P}\{70 < T < 130\} \geq ??$