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Name: _____

ISyE 2027
Test 3

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) What would be a reasonable guess as to the distribution of the following random variable? (a) The number of dogs seen at a vet clinic in 2013 until seeing a dog with meningitis. (b) The number of dogs seen at a vet clinic during May that had been hit by a car. (c) The length of time until a vet tech gets bitten by a cat.

2. (30 points) A lake contains 25 red fish, 25 blue fish, 25 green fish and 25 yellow fish. A net randomly selects 6 fish from the lake. (a) What is the probability that there are 2 of one color, 2 of another, and 2 of a third color? (b) What is the probability that there is 3 of one color, 2 of another color, and 1 of a third color? (c) What is the probability that all six fish are the same color?

3. Suppose the demand D for the weekend has p.m.f. $\mathbb{P}\{D = k\} = (5-k)/15$ for $k = 0, \ldots, 5$. (a) Compute the mean of D. (b) Compute the variance of D. (c) Assume there are only two items in inventory at the beginning of the weekend. What is the expected number of items sold?

4. Suppose a miniload miniload system with a rack that is 20 meters long and 20 meters high. There are two motors. One motor moves the s/r machine horizontally at 2 meter per second, and the other moves the s/r machine vertically at the same speed. The two motors can operate simultaneously so the time T from the origin to a random location (X, Y) is whichever time is longer between the time to move vertically and the time to move horizontally. Assume that 25% of the items cause 90% of the activity and these items are stored in the lower left quarter of the rack near the i/o point. (a) Express the travel time T as a function of (X, Y). (b) Determine the c.d.f. of T. (c) What is the marginal p.d.f. of X?

5. (30 Points) Let X_1, X_2, X_3 be three random variables. Suppose all three have mean μ and variance 1. The sample mean is $Y = (X_1 + X_2 + X_3)/3$. (a) Can you compute the mean of Y? If so, what is it? If not, why not? (b) If we assume that the three random variables are independent, what would the variance of Y be? (c) If the covariance between X_i and X_j were actually $(1/3)^{|i-j|}$, then what would the variance really be?

6. Suppose the weight of a bolt is 1 gram and the standard deviation is .15 grams. Consider the combined weight of 100 bolts, and assume that the weights of the bolts are i.i.d. What is the mean of the combined weight? (b) What is the standard deviation of the combined weight? (c) What is the probability that the combined weight is between 97 and 103 grams?