

Name: \_\_\_\_\_

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ISYE 2027  
Test 3

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like  $\binom{52}{5}$  and  $e^{-2}$  in your answers.

1. (30 points) Suppose you are dealt 6 cards from a standard deck. Compute the probability of being dealt (a) three pairs including one pair of aces, (b) one pair, three of a kind and an ace. (c) the ace, king, queen, jack, 10, and 9 all in hearts.
2. (30 points) Let  $X$  and  $Y$  be independent Bernoulli random variables with mean  $1/2$ . (a) Compute  $\mathbb{P}\{|X - Y| = 0\}$  (b) Compute  $\mathbb{P}\{X + Y = 0, |X - Y| = 0\}$ , (c) Are  $X + Y$  and  $|X - Y|$  independent?
3. (30 points) Publix received 100 ducks. The weights of the ducks are believed to be i.i.d. with mean 6 pounds and variance 4 pounds<sup>2</sup>. (a) What is the approximate probability that the total weight of the ducks (a) exceeds 600 pounds? (b) is less than 560 pounds? (c) exceeds 660 pounds?
4. (30 points) Let  $X$  and  $Y$  be independent Poisson random variables with means 9 and 16, respectively. Compute (a)  $\mathbb{E}[\min\{X, 2\}]$ , (b)  $\text{Var}(X + Y)$  (c)  $\text{Cov}(X + Y, X)$ .
5. (30 points) Suppose we have a miniload system with a storage rack that is 60 meters long and 20 meters high. There are two motors moving the s/r device from the bottom front of the rack to a random location to retrieve a tray. One motor moves the s/r machine horizontally at 3 meters per second, while the other motor moves it vertically at 1 meter per second. The two motors can operate simultaneously so the time  $T$  from the origin to a random location  $(X, Y)$  is whichever time is longer of the time to move horizontally and to move vertically. Assume that 25% of the items cause 80% of the activity, and these items are stored in region closest to the bottom front of the rack. The travel time  $T$  is some function  $g(X, Y)$ . (a) What is  $g(X, Y)$ ? (b) What is  $\mathbb{P}\{T \leq 5 \text{ seconds}\}$ ? (c) What is  $\mathbb{P}\{T \leq 5 \text{ seconds} \mid T \leq 10 \text{ seconds}\}$ ?