

Name: _____

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ISyE 2027
Test 3

Calculators, notes, and books are not allowed. Please show your work in your bluebook and transfer your answers to the back of this sheet. Put your name on everything and hand in both the bluebook, test, and answer sheet. The test starts at 9:00 and ends at 10:30. Please stop working when time is up.

- (30 points) What would be a reasonable guess for the distribution of the following random variables. (a) the total weight of 12 cups (scooped from a bag) of flour? (b) the number of eggs cracked until a bad egg is encountered? (c) the number of raisins in an oatmeal cookie? (d) whether the souffle rises? (e) the number of phone calls received during the next 3 hours of baking? (f) the length of time until the first phone call?
- A pinochle deck has 48 cards. The ranks are Ace, King, Queen, Jack, 10, and 9. There are the same four suits, but there are two cards of each rank and suit (two Aces of hearts, two Aces of spades, and so on). Suppose we are playing playing poker with a pinochle deck. Assuming we are dealt five cards, what is the probability of (a) a full house? (b) two pairs? (c) 5 aces?
- (30 points) A friend has two cell phones. The length of time until the next call on the cell phone for work is exponentially distributed with a mean of 30 minutes. The length of time until the call on the personal cell phone is exponentially distributed with a mean of 15 minutes. (a) What is the probability that the work phone rings before the personal phone? (b) What is the expected length of time until a phone rings. (c) It has been 25 minutes since someone has called on the work phone. What is the expected time until the next call on the work phone?
- (30 points) Suppose you play the following game. You roll a die once. Let X be the value of the first roll. Once you know X , you must decide whether to roll a second time or not. If you roll a second time, let Y denote the value of the die on the second roll. If you roll only once, your reward is $R = X$. If you roll twice, your reward is the average of the two rolls; that is, $R = (X + Y)/2$. (a) For what values of X is it optimal to roll a second time? (b) Assuming you play optimally, compute $E[R | X = k]$ for $k = 1, \dots, 6$. (c) Compute $E[R]$.
- (30 points) Suppose (X, Y) has joint p.d.f. $f_{(X,Y)}(s, t) = ce^{-(10s+9t)}$ for $s \geq 0$ and $t \geq 0$. Compute (a) c , (b) $\Pr\{X > 1\}$, and (c) $\Pr\{X = Y\}$. (Hint: you can either simply compute the desired quantities, or you could look at the joint p.d.f. for a few minutes, hoping that you will see what is going on in this problem.)
- (30 points) A worker has to retrieve 100 items. The time needed to retrieve a single item has a mean of 2.7 minutes and a variance of 9 minutes². Assume that the times to retrieve the 100 items are independent and identically distributed. (a) What is the average time in minutes needed to retrieve all 100 items? (b) What is the standard deviation of the total time (including specifying the appropriate units) needed to retrieve all 100 items? (c) What is the probability that the worker needs more than 5 hours to retrieve all 100 items?