

ISyE 2027  
Test 2

Calculators, notes, and books are not allowed. Please show your work in the bluebook and transfer your answers to the back of this sheet. Put your name on everything and hand in both the bluebook, test, and answer sheet.

- (30 points) Leave your answers in terms of  $\binom{n}{k}$ . Suppose we are dealt 9 cards from a poker deck without any aces (so there are only 48 cards in the deck). (a) What is the probability of 3 triples? (b) What is the probability of a flush (all 9 cards in the same suit)? (c) What is the probability of getting 4 of one kind, 3 of another kind, and a pair?
- (30 points) Suppose someone takes route A forty percent of the time, and otherwise takes route B. Today, the person will be late with probability  $1/100$  on route A, but will be late on route B with probability  $9/10$  due to an accident. (a) What is the probability that the person is late? (b) Given that the person is late, what is the probability that they took route B? (c) Given that the person took route B, what is the probability that they took route A?
- (30 points) Let  $X$  have p.m.f.  $\Pr\{X = k\} = c(2 + k)$  for  $k = -1, 0, 1$ . (a) What is the mean of  $X$ ? (b) What is the variance of  $X$ ? (c) What is  $\Pr\{X > 0\}$ ?
- (30 points) Let  $Y$  have p.d.f.  $f(s) = s/4 + 1/2$  for  $-1 \leq s \leq 1$ . (a) What is  $\Pr\{Y = -1\}$ ? (b) What is  $E[Y]$ ? (c) What is  $\Pr\{Y > 0\}$ ?
- (30 points) Turtles A and B are racing. The length of time until A finishes the race is exponentially distributed with mean 30 minutes, and for B, exponentially distributed with mean 20 minutes. Assume that their two times are independent. (a) What is the probability that A takes longer than 1 hour? (b) What is the probability that A wins? (c) What is the expected time of the winning turtle in hours?
- (30 points) Suppose  $X$  has mean 2 and variance 9. Suppose  $Y$  has mean 4 and variance 25. Suppose  $X$  and  $Y$  have covariance 10. (a) What is the mean of  $4X$ ? (b) What is the variance of  $4X$ ? (c) What is the variance of  $X + Y$ ?
- (30 points) Suppose we have a miniload system with a rack 40 meters long and 20 meters high. There are two motors. One motor moves the s/r machine horizontally at 2 meters per second, while the other motor moves it vertically at 1 meter per second. The two motors can operate simultaneously so the time  $T$  from the origin to a random location  $(X, Y)$  is whichever time is longer of the time to move horizontally and to move vertically. Assume that the random location visited is uniformly distributed over the rack. (a) The travel time  $T$  is some function  $g(X, Y)$ . What is  $g(X, Y)$ ? (b) What is  $F_T(x)$ , i.e., the c.d.f. of  $T$ ? (c) What is the marginal density function of  $X$ ?