Name: _____

R. D. Foley
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ISyE 2027
Test 3

Calculators, notes, and books are not allowed. Please work in the bluebook and transfer your answers to the back of this sheet. Put your name on everything and hand in both the bluebook, test, and answer sheet.

1. (30 points) Let X have mean 3 and standard deviation 4. Let $Y = 3X + 1$. (a) What is $\mathrm{E}[Y]$? (b) What is $\mathrm{Var}[Y]$? (c) What is $\mathrm{Cov}[X, Y]$?

2. (30 points) Suppose $\Pr\{X = k\} = (3 - k)/6$ for $k = 0, 1, 2$. (a) What is the mean of X? (b) What is the variance of X (c) What is the $\mathrm{E}[g(X)]$ where $g(0) = 2$, $g(1) = 1$, and $g(2) = 3$?

3. (30 points) The squared coefficient of variation is defined as $\mathrm{Var}[X]/(\mathrm{E}[X]^2)$. What is the squared coefficient of variation of an exponential random variable with mean $1/\mu$? What is the name of the most important property of the exponential distribution?

4. (30 points) Suppose you are dealt a hand of five cards from a standard deck. Let X denote the number of aces, and Y, the number of red Kings in the hand. What is the joint probability mass function of (X, Y), i.e., $\Pr\{X = i, Y = j\}$?

5. (30 points) A randomly selected part could have been in several different factories and could contain defects. Let X denote the factory where it was produced, and let Y denote the number of defects. Assume that $\Pr\{X = i, Y = j\} = (3 - i - j)/9$ for $i = 0, 1$ and $j = 0, 1, 2$. Also assume that each defect costs \$100 to repair. (a) What is the marginal probability mass function of Y? (b) What is the conditional probability mass function of the number of defects given that the part was produced in Factory 0? (c) What is the expected cost due to repairing defects?

6. (30 points) Assume that Pat arrives at the bus stop at some time U, which is uniformly distributed between time 0 and time 1, and waits for a bus. The first bus arrives at time T which is exponentially distributed with mean $1/\mu$. Assume that U and T are independent. What is the probability that Pat catches the first bus? Hint: one way, but not the only way, to start solving this problem would be the following:
$$\Pr\{U < T\} = \int_0^\infty \Pr\{U < T \mid U = s\} f_U(s)\, ds.$$

7. (30 points) A farmer buys 100 bales of hay. A bale of hay has an average weight of 50 pounds and variance 100 pounds^2. What is the probability that the farmer purchased less than 4700 pounds of hay?