ISyE 2027D
Test 3

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose $P\{X = k\} = (4 - k)^2/14$ for $k \in \{1, 2, 3\}$.

   (a) Compute the mean of X.
   (b) Compute $P\{X = 1 \mid X \leq 2\}$.
   (c) Compute $E[X \wedge 2]$.

2. (30 points) Suppose X has c.d.f. $F(t) = s^3/8$ for $0 \leq s \leq 2$.

   (a) Compute the mean of X.
   (b) Compute the variance of X.
   (c) Compute $E[1/X^2]$.

3. (30 points) Suppose $P\{X = i, Y = j\} = j/9$ for $i = 0, 1, 2$ and $j = 1, 2$. Compute

   (a) $P\{X + Y = 2\}$,
   (b) $P\{Y = 2 \mid X = 2\}$
   (c) $E[|X - Y|]$.

4. (30 points) Consider a random circle with radius R meters, circumference $C = 2\pi R$ meters and area $A = \pi R^2$ meters^2 where R is exponentially distributed with mean $1/\sqrt{\pi}$ meters.

   (a) Compute $\text{Cov}(R, C)$.
   (b) Compute $P\{A > 9\}$.
   (c) Compute $P\{A > 9 \mid A > 4\}$.

5. (30 points) Consider a miniload system with a storage rack that is 40 meters long and 20 meters high. Assume that there are two motors: one that moves the s/r machine horizontally at speed 2 meters per second, and one that moves the s/r machine vertically at speed 1 meter per second. Let T be the time in seconds for the s/r machine to move from the origin to a random point (X, Y) that is uniformly distributed over the rack face.

   (a) Define T as a function of X and Y.
   (b) Compute $P\{T \leq 5\}$.
   (c) Compute $E[T]$.

6. (30 points) Let N be the number of cars that fail out of the next 36 cars tested. Assume the results of the 36 emission tests are i.i.d. random variables.

   (a) If on average 1 car in 3 fails, approximate the probability that N would be greater than or equal to 15?
   (b) If on average 1 car in 18 fails, approximate the probability that none of the 36 fails.
   (c) Can you find an upper bound on the probability that the total time to test the 36 cars takes 10 hours or more assuming that the average test time for one car is 15 minutes?

   In addition to being allowed to leave your answer in terms described in the instructions at the beginning, you may also leave your answer in terms of $P\{a < Z \leq b\}$ for suitable values a and b where Z is a standard Gaussian random variable. However, please do not include any summations in your answers to these questions.