ISyE 2027
Test 2

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points)

   My friend Joel has two children. In the following, you may assume that genders are equally likely, so are the 7 days of the week and the two are independent.

   (a) Suppose I know that the elder child is female and born on Monday. What would the (conditional) probability be that both children are female?

   (b) Suppose I only know that at least one of the children is female. What would the (conditional) probability be that both are female?

   (c) Suppose I know that at least one of the children is female and born on Monday. Now what would the (conditional) probability be that both are female?

2. (30 points) Suppose $P\{X = k\} = k^2/14$ for $k \in \{1, 2, 3\}$.

   (a) Compute the mean of X.

   (b) Compute $P\{X = 1 \mid X \leq 2\}$.

   (c) Compute $E[X \wedge 2]$.

3. (30 points) Suppose X has c.d.f. $F(t) = s^2/4$ for $0 \leq s \leq 2$.

   (a) Compute the mean of X.

   (b) Compute the variance of X.

   (c) Compute $E[1/X^2]$.

4. (30 points) A bag contains 3 coins. Two of the coins are fair, and one is unfair. The unfair coin comes up heads with probability 5/6.

   (a) A coin is selected at random and tossed. What is the probability of that it comes up heads?

(b) Given that the coin came up heads, what is the probability that it is the unfair coin?

(c) The coins are placed back in the bag, and the bag shaken. Two coins are selected at random. What is the probability that both are fair?

5. A mushroom hunter travels a random distance of L kilometers before finding a patch of morels. Assume that the mean of L is 3 kilometers, and that the mushroom hunter walks at 5 kilometers per hour. The mushroom hunter needs an additional 1/5th of an hour to collect the morels. Let T be the total time to locate the morel patch and collect the mushrooms (but not including returning).

(a) Can you get a good upper bound on $P\{T \geq 4\}$?

(b) If L were exponentially distribution and the mushroom hunter has traveled 2.3 kilometers, how many more kilometers does the hunter expect to travel before discovering the morels?

(c) If L were exponentially distributed, What would be the probability that L is greater than 6 kilometers?

6. Suppose Connor walks 100 paces, and his paces are identically distributed with mean 2.5 feet and standard deviation 1/2 foot.

(a) If the lengths of his paces are independent, what is the approximate probability that the distance covered is between 240 and 260 feet?

(b) Suppose the lengths of his paces are dependent and positively correlated. Would the variance of the distance covered be larger or smaller than in the i.i.d. case?

(c) Would the probability that the distance covered in the positively correlated case be larger or smaller than for the i.i.d. case?