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ISyE 2027B
Test 2

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Let X have p.m.f. $\mathbb{P}\{X = k\} = |k|/4$ for $k = -1, 1, 2$. Compute:

   (a) the mean of X,

   (b) the standard deviation of X,

   (c) $\mathbb{E}[\cos((X \wedge 1/2)\pi)]$ where $a \wedge b$ denotes the minimum of a and b.

2. (30 points) Suppose that the c.d.f. of Y is $F(s) = s^3$ for $0 < s < 1$. Compute the following:

   (a) the mean of Y,

   (b) the median of Y,

   (c) the variance of Y.

3. (30 points) You purchase 2 items at \$500 per item, and you try to sell them for \$2,000 per item. Let D denote the demand. You can only sell items that you have on hand (no backorders), so if the demand exceeds 2, those sales are lost. The net revenue is the gross revenue from sales minus the cost of purchasing the two items.

   (a) Compute $\mathbb{P}\{D \geqslant 2\}$. (At most half credit if your answer contains an infinite summation.)

   (b) Write down an expression for the net revenue as a function of D.

   (c) Compute the expected net revenue assuming the demand has a Poisson distribution with mean λ.

4. (30 points) Suppose we take 2 standard decks and shuffle them together so there are a total of 104 cards. Compute the probability of the following assuming we are dealt 5 cards:

   (a) a full house (that is, 3 of one kind and 2 of another kind),

   (b) two pairs (that is, 2 of one kind and 2 of another kind),

   (c) 5 hearts.

5. (30 points) A robot travels a random distance D to retrieve an item and then returns where D has mean 8 minutes and variance 9 minutes^2. The time to pick up the item is a constant c minutes. Assume that the robot moves at a constant speed s meters per minute. Let T be the total round trip travel time plus pick-up time.

   (a) What is the mean of T?

   (b) What is the variance of T?

   (c) What is an upper bound on $\mathbb{P}\{|D - 8| \geqslant 6\}$?