

R. D. Foley
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Name: _____

ISyE 2027
Test 1

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose we are dealt 5 cards from a well-shuffled standard deck that has had all of the hearts removed. Thus, there are only 3 suits and 39 cards. What is the probability of receiving (a) 2 pairs? (b) a full house, which is 3 of one kind and 2 of another? (c) three aces?
2. (30 points) Suppose X has p.d.f.

$$f(s) = 3s^2, 0 \leq s \leq 1$$

Compute (a) $\mathbb{E}[X]$, (b) $\mathbb{E}[1/X]$, and (c) the median of X .

3. (30 points) Suppose X has c.d.f.

$$F(t) = t^2, 0 \leq t \leq 1.$$

- (a) Is $\mathbb{E}[X]$ larger or smaller than $1/2$? (b) Find the p.d.f. of X . (c) Explicitly define $g(U)$ where U is a uniform $(0,1)$ r.v. so that $g(U)$ has c.d.f. F .
4. (30 points) Suppose the times between taxis are exponentially distributed with mean 4 minutes. Compute (a) What is the probability that you wait more than 2 minutes for a cab? (b) If you've already waited 3 minutes, how much longer do you expect to wait for a cab? (c) What is the variance of $2X + 25$.
5. (30 points) Items are being inspected. Whether an item fails inspection or not is independent of the other items. Items fail inspection with probability $1/5$ th. Let N be the number of items that fail out of the first 25 items inspected. (a) What is the exact probability that N is less than or equal to 7? (Leave your answer as a summation). (b) What is the variance of N ? (c) What is the value of x so that $\mathbb{P}\{N \leq 7\} \approx \mathbb{P}\{Z \leq x\}$ where Z is a standard Gaussian random variable? (Use a continuity correction if appropriate.)
6. Let $X = 5Z + 7$ where Z has a standard Gaussian (a.k.a. Normal) distribution. (a) What is the distribution of X ? (b) What is $\mathbb{P}\{X \leq 12\}$? (c) What is $\mathbb{P}\{|X - 5| > 15\}$?