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ISyE 2027
Test 2

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) What would be a reasonable distribution for each of the following random variables? (a) The number of typos in our textbook. (b) The number of pages until the first page with a typo? (c) The number of typo free pages out of the first 100 pages.

2. (30 points) Suppose X has p.d.f.

$$f_X(s) = 2s, \text{ for } 0 < s < 1$$

Compute (a) $\mathbb{E}[X]$, (b) $\text{Var}[X]$, and (c) $\mathbb{P}\{X > 1/2\}$.

3. (30 points) Suppose we construct a square where the length of each side is a random variable L feet where L has p.d.f.

$$f_L(s) = s, \text{ for } 0 < s < 2.$$

Let A the area of the square, and $f_A(s)$, the p.d.f. of A. Compute (a) $\mathbb{E}[A]$, (b) $\mathbb{P}\{A \leqslant 2 \text{ square feet}\}$, and (c) $f_A(s)$.

4. (30 points) Let X have a binomial distribution with parameters $n = 100$ and $p = 1/5$. Let Y have a normal distribution with the same mean and variance as X. (a) What is the mean and variance of Y? (b) The $\mathbb{P}\{X \leqslant 56\}$ is approximately equal to $\mathbb{P}\{Y \leqslant \text{what}\}$? (Use a continuity or histogram correction if appropriate.) (c) Compute $\mathbb{P}\{\lfloor X/2 \rfloor = 0\}$.

5. (30 points) Let Z have a standard normal distribution. Let Y have a Gaussian distribution with mean 5 and variance 49. Compute (a) $\mathbb{P}\{Z > 2\}$, (b) $\mathbb{P}\{Y \leqslant 19\}$, and (c) the variance of $(Y - 5)/7$.

6. (30 points) Suppose the service time S of a customer is exponentially distributed with mean 5 minutes. (a) What is the probability that the service time takes longer than 2 minutes? (b) Given that the service time is longer than 4 minutes, what is the expected remaining service time? (c) Compute the Laplace-Stieltjes transform of S; that is, compute $\mathbb{E}[e^{-tS}]$.

7. (30 points) Suppose a picker stands at the beginning of a 100 foot aisle. The picker walks to a random location L that is uniformly distributed over the length of the aisle. Suppose the picker walks at 5 feet per second to the location L and back. At the location, the picker takes 15 seconds to remove an item from the rack. (a) What is an expression for the total round trip time R including the 15 second pick time? (b) Compute $\mathbb{P}\{R \leqslant 25 \text{ seconds}\}$. (c) What is the variance of R?