

Name: \_\_\_\_\_

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ISYE 2027  
Test 2

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like  $\binom{52}{5}$  and  $e^{-2}$  in your answers.

- (30 points) Suppose you are dealt 5 cards from a standard deck. Compute the probability of being dealt (a) 2 cards from one suit, 2 cards from another suit, and one card from a 3rd suit; (b) 3 cards from one suit, and 2 cards from another suit; (c) the “dead man’s hand,” which is usually assumed to be a pair of aces and a pair of eights.
- Plant A produces 60 percent of the frying pans, and Plant B produces the rest. The defect rate at Plant A is one percent. The defect rate at B is 2 percent. (a) What is the probability that a randomly selected pan is defective and produced at A? (b) What is the probability that a randomly selected pan is defective? (c) Given that the pan is defective, what is the probability that it came from B?
- (30 points) Suppose  $X$  has mean 5 and variance 16. Let  $Y = -3X + 6$ . (a) What is the mean of  $Y$ ? (b) What is the variance of  $Y$ ? (c) What is a good (that is, least) upper bound for  $\mathbb{P}\{|X - 5| \geq 20\}$ ?
- (30 points) Suppose we have 5 items in stock in the morning. The demand  $D$  during the day for that item has p.m.f.  $\mathbb{P}\{D = k\} = (1 - p)^{k-1}p$  for  $k = 1, 2, \dots$ . Let  $N$  be the number of those items sold during that day. (a) Give an expression for  $N$  as a function of  $D$ . (b) What is  $\mathbb{P}\{N = 2\}$ ? (c) What is  $\mathbb{P}\{N = 5\}$ ?
- (30 points) Suppose a picker standing at the beginning of a 50 meter aisle needs to go to a random location  $L$ . The picker moves at speed 2 meters per second and needs 10 seconds to pick the item from the rack. Let  $R$  be the round trip time to  $L$  and back including the pick time. The p.d.f of  $L$  is  $f(s) = 1/25 - s/1250$  for  $0 \leq s \leq 50$  meters. The mean of  $L$  is  $50/3$  meters, and the variance of  $L$  is  $1250/9$  meters<sup>2</sup>. Compute (a)  $\mathbb{P}\{L \leq 25\}$ , (b) Compute  $\mathbb{P}\{R > 35\}$ , and (c) the squared coefficient of variation of  $L$ .
- (30 points) Let  $X$  be the number of times that a fair coin lands heads in 100 tosses. Let  $Y$  be a normal random variable with the same mean and variance as  $X$ . (a) What is the standard deviation of  $X$ ? (b) If we used the continuity correction,  $\mathbb{P}\{X = 43\}$  would be approximated by  $\mathbb{P}\{a \leq Y \leq b\}$  where  $a + b$  equals what? (c) Compute  $\mathbb{P}\{Y \geq 60\}$ .
- Suppose we take  $N$  shots at a target. Each shot hits the target with probability  $z$  independently of the other shots. What is the probability that we hit the target on all  $N$  shots where  $N$  has distribution (a) Bernoulli with parameter  $p$ , (b) geometric with parameter  $p$ , and (c) Poisson with parameter  $\lambda$ ?