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Name: _____

ISyE 2027
Test 1

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like 525 and e^{-2} in your answers.

1. (30 points) What would be a reasonable guess as to the distribution of the following random variable? (a) The number of chocolate chips in the tenth chocolate chip cookie, (b) The number of chocolate chip cookies received until receiving a cookie with exactly 10 chocolate chips, (c) The number of chocolate chip cookies out of the first 10 cookies that have exactly 10 chocolate chips.

2. (30 points) Suppose we are dealt 6 cards from a well-shuffled standard deck. What is the probability of (a) 4 of one kind and 2 aces? (b) three pairs? (c) three pairs including a pair of aces (that is, a pair of aces and 2 other pairs)?

3. (30 points) Suppose X has p.m.f.

$$X = k = \{\ 4\ 9/50 for$$

k = -1/7
$1/50 for k = 7$ Compute (a) $[X]$, (b) $[X]$, and (c) $[\max(X, 0)]$.

(30 points) Suppose the probability that our team beats A is 6/10, but the probability that our team beats B is 9/10. The probability that we play A first is 2/3; otherwise, we play B first. (a) What is the probability that we win the first game? (b) Given that we win the first game, what is the probability that we played B in that game? (c) What is the probability that we play A first and lose?

(30 points) Suppose the number of jobs process is X where X has a Poisson distribution with mean 4. Compute (a) the probability that there is one job to process, (b) the probability that there is one job given that there are at most 3 jobs, and (c) the variance of the total time to process all X jobs assuming that there is a setup time of 25 minutes followed by 3 minutes per job.

(30 points) (a) According the APS Investigative Report, a class should be flagged with probability 1/740. Suppose we have 2220 classes that are independently flagged, each with probability 1/740. Accurately approximate the probability that exactly one of the 2220 classes is flagged. Your answer may include expressions of the form e^x, but your answer should not include expressions of the form nk or factorials. (b) Suppose X has mean 0 and variance 1. Can you obtain a good upper bound on the $|X| \geq 7$? (c) Suppose $Y \geq 0$ and the $[Y^3] = 2$. Can you obtain a good upper bound on $Y \geq 3$?