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ISyE 2027D
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose $\mathbb{P}(A) = 3/11$, $\mathbb{P}(B) = 4/11$, and $\mathbb{P}(AB) = 1/11$. Determine the following.

    (a) $\mathbb{P}(A^c)$,

    (b) $\mathbb{P}(A \cup B)$

    (c) $\mathbb{P}(AC)$ where $C = B^c$.

2. (30 points) Suppose we are dealt 8 cards from a standard deck. What is the probability of each of the following hands?

    (a) 4 of one rank, 3 of another rank, and one of a third;

    (b) 4 pairs;

    (c) 4 black cards and 4 diamonds.

3. (30 points) Suppose X has mean 4 and variance 9. Suppose $Y = 2 - 5X$

    (a) What is the mean of Y?

    (b) What is the standard deviation of Y?

    (c) What is the squared coefficient of variation of X (not Y)?

4. (30 points) $\mathbb{P}\{X = k\} = ck^2$ for $k = -2, -1, 1, 2$ and $Y = \max(0, X)$ where max means maximum. Thus, $\max(0, -3) = 0$, and $\max(0, 3) = 3$. Compute the following

    (a) $\mathbb{E}[X]$,

    (b) $\mathbb{E}[\max(0, X)]$,

    (c) $\mathbb{P}\{Y = 0\}$.

5. (30 points) Suppose $\mathbb{P}\{X = k\} = cq^k$ for $k = 1, 2, \ldots$ and zero otherwise. Compute each of the following. (You will receive at most half points for a part if your answer for that part contains an infinite summation.)

    (a) c,

    (b) $\mathbb{P}\{X \geqslant 2\}$,

    (c) $\mathbb{E}[z^X]$ where $0 < z < 1$.

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