R. D. Foley   Name: _____
February 17, 2017

ISyE 2027B
Test 1

Calculators, notes, and books are not allowed. Put your name on both sides of this sheet. Please stop when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) Suppose $\mathbb{P}(A) = 3/10$, $\mathbb{P}(B) = 4/10$, and $\mathbb{P}(AB) = 1/10$. Determine the following.

   (a) $\mathbb{P}(A^c)$,

   (b) $\mathbb{P}(A \cup B)$

   (c) $\mathbb{P}(AC)$ where $C = B^c$.

2. (30 points) Suppose we are dealt 6 cards from a standard deck. What is the probability of each of the following hands?

   (a) 3 of one rank, 2 of another rank, and one of a third;

   (b) 3 pairs;

   (c) 3 black cards and 3 diamonds.

3. (30 points) Suppose X has mean 2 and variance 16. Suppose $Y = 4 - 3X$

   (a) What is the mean of Y?

   (b) What is the standard deviation of Y?

   (c) What is the squared coefficient of variation of X (not Y)?

4. (30 points) $\mathbb{P}\{X = k\} = c|k|$ for $k = -2, -1, 1, 2$ and $Y = \max(0, X)$ where max means maximum. Thus, $\max(0, -3) = 0$, and $\max(0, 3) = 3$. Compute the following

   (a) $\mathbb{E}[X]$,

   (b) $\mathbb{E}[\max(0, X)]$,

   (c) $\mathbb{P}\{Y = 0\}$.

5. (30 points) Suppose $\mathbb{P}\{X = k\} = c2^k/k!$ for $k = 0, 1, 2, \ldots$ and zero otherwise. Compute each of the following. (You will receive at most half points for a part if your answer for that part contains an infinite summation.)

   (a) c,

   (b) $\mathbb{P}\{X \geqslant 2\}$,

   (c) $\mathbb{E}[1/(X + 1)]$.

1