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ISyE 2027
Test 1

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

1. (30 points) What would be a reasonable distribution for each of the following random variables? (a) The number of eggs in the next carton of eggs that exceed 60 grams in weight. (b) The number of cartons of eggs I buy until I get a carton of eggs weighing more than 750 grams. (c) Whether the next carton of eggs has more blue/green eggs than brown eggs.

2. (30 points) Suppose we remove all 13 hearts from a standard deck so that we have only 39 cards in the deck. Suppose we are dealt 5 cards. What is the probability of (a) a full house (3 of one kind and 2 of another)? (b) two pairs? (c) a flush (all 5 cards in one suit).

3. (30 points) Suppose X has p.m.f.

$$\mathbb{P}\{X = k\} = \begin{cases} 1/6 \text{ for k} = -2 \\ 2/6 \text{ for k} = 0 \\ 3/6 \text{ for k} = 1 \end{cases}$$

   Compute (a) $\mathbb{E}[X]$, (b) $Var[X]$, and (c) $\mathbb{P}\{X = 1 \mid X > -1\}$.

4. (30 points) A manufacturing company receives a particular type of part from two different suppliers. Supplier A provides three fifths of the parts (3/5), and Supplier B the rest. One percent of Supplier A's are defective, while two percent of B's are defective. (a) What is the probability that a randomly selected part is defective and provided by A? (b) What is the probability that a randomly selected part is defective? (c) Suppose a randomly selected part is defective. What is the probability that it was made by A?

5. (30 points) A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and $10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 3. (a) What is the probability of 2 major snowstorms? (b) What is the standard deviation of the number of major snowstorms? (c) What is the expected amount of money that the company receives from the policy during the year? (Do not leave as an infinite summation.)

6. (30 points) Suppose $X \geqslant 0$ and has mean 10. (a) Can you get a good upper bound on $\mathbb{P}\{X \geqslant 25\}$? (b) Suppose it's also true that $X \geqslant 5$. Can you get a better bound on $\mathbb{P}\{X \geqslant 25\}$ (c) If the variance of X were 7, what would the variance of $3X + 5$ be?

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