Name:

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ISyE 2027 Test 1

Calculators, notes, and books are not allowed. Put your name on back and front of this sheet. Please stop working when time is up. You may leave terms like $\binom{52}{5}$ and e^{-2} in your answers.

- (30 points) A tick is a small insect. A lone star tick is a particular type of tick. Tick bites sometimes transmit various diseases including Lyme disease and Erlichiosis. What would be a reasonable guess for the distribution of each of the following: (a) The number of tick bites that you receive during May 2012. (b) The number of tick bites that you receive until being bitten by a lone star tick. (c) Out of the next 5 ticks that bite you, whether any of those 5 ticks were infected with Erlichiosis or not.
- 2. (30 points) Suppose the event A has probability 6/10, the event B has probability 8/10, and the event AB has probability 5/10. What is the probability of (a) A^{c} ? (b) A given B? (c) AC where $C = A^{c}$?
- 3. (30 points) Suppose Z has p.m.f.

$$\mathbb{P}\{\mathsf{Z} = \mathsf{k}\} = \begin{cases} 4/7 & \text{for } \mathsf{k} = 0, \\ 3/7 & \text{for } \mathsf{k} = 7, \\ 0 & \text{otherwise} \end{cases}$$

Compute (a) the mean, (b) the variance, and (c) a median of Z.

- 4. (30 points) Suppose X has mean 4 and variance 9. Let Y = -3X + 6. (a) What is the mean of Y? (b) What is the variance of Y? (c) What is a good (that is, least) upper bound for $\mathbb{P}\{|X 4| \ge 15\}$?
- 5. (30 points) Suppose X has p.m.f. $\mathbb{P}\{X = k\} = e^{-\lambda}\lambda^k/k!$ for k = 0, 1, 2, ...What is (a) $\mathbb{E}[X]$, (b) $\mathbb{P}\{X = 0 \mid X \leq 1\}$, (c) $\mathbb{E}[X(X - 1)(X - 2)]$.
- 6. (30 points) Suppose I am tossing markers at the basket. Assume that the results of the tosses are independent and that I am successful on each toss with probability p. Let Y be the number of tosses until the SECOND success, so $Y \ge 2$. Compute (a) $\mathbb{P}{Y = 2}$, (b) $\mathbb{P}{Y = 50}$, and (c) $\mathbb{E}[Y | A]$ where A is the event that I am successful on the first toss.