

ISyE 2027  
Test 1

Calculators, notes, and books are not allowed. Please show your work in the bluebook and transfer your answers to the back of this sheet. Put your name on everything and hand in both the bluebook, test, and answer sheet.

- (30 points) Suppose  $\Pr(A) = .3$ ,  $\Pr(B) = .6$ , and  $\Pr(A \cap B) = .2$ . Compute (a)  $\Pr(\bar{B})$ , (b)  $\Pr(A \cup B)$ , (c)  $\Pr(B | A)$ , (d)  $\Pr(\bar{B} | A)$ , and (e)  $\Pr(A | \bar{B})$ .
- (30 points) Suppose we flip a coin twice. (a) What is the probability of two heads? (b) What is the probability of two heads given the first toss is heads? (c) What is the probability of two heads given that at least one of the tosses is heads?
- (30 points) Let  $X$  be a discrete random variable with probability mass function  $\Pr\{X = k\} = c(5/6)^{k-1}$  for  $k = 1, 2, \dots$ . (a) Determine  $c$ . (b) Compute  $\Pr\{X = 1 | X \leq 2\}$ . (c) Compute  $\Pr\{X \leq 2 | X = 1\}$ .
- (30 points) Leave your answer to in terms of  $\binom{n}{k}$ . Suppose we are dealt 4 cards from a well-shuffled standard poker deck. (a) What is the probability of a pair of aces (and nothing better)? (b) What is the probability of 2 pairs? (c) What is the probability of 3 of a kind?
- (30 points) A particular component is used in assembling products. We obtain 3/4 of these components from Supplier A and the rest from Supplier B. One percent of the components from Supplier A are defective, while two percent of Supplier B's are defective. (a) What is the probability that a randomly selected component is defective? (b) Given that a randomly selected component is defective, what is the probability that it came from Supplier A? (c) Is the quality of the component and the supplier independent?
- (30 points) Suppose we select and inspect parts from Supplier A, one at a time. Let  $X$  be the number of parts selected until the first defect. Let  $Y$  be the number of parts selected until the 3rd defective part. (a) What is the probability mass function of  $X$ . (b) What is the name of the distribution of  $X$ ? (c) What is the probability mass function of  $Y$ ?
- (1 point) Bonus: Man o'War was only beaten once. What was the name of the horse that upset Man o'War?