Name: ______________________________
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ISyE 3027a
Test 2

[30] 1. Suppose $\Pr\{X = i\} = c(i + 2)$ for $i = -1, 0, 1$ and zero otherwise.

(a) Determine c. ______________________________

(b) Compute $\Pr\{X \leq 1/2\}$. ______________________________

(c) Compute $\Pr\{X = 0 | X = 0\}$. ______________________________

(d) Compute the mean of X. ______________________________

(e) Compute the variance of X. ______________________________

(f) Suppose a bonus is received if $X \neq -1$ of 1000 dollars. What is the expected bonus? ______________________________

[30] 2. Let Y be a random variable with probability density function $f(t) = ct$ for $0 \leq t \leq 2$. Compute the following:

(a) c

(b) $\Pr\{Y \geq 1\}$

(c) $\Pr\{Y > 3/2 \mid Y > 1\}$

(d) The mean of Y

(e) The variance of Y

(f) $\Pr\{Y = 1\}$

[30] 3. Determine whether the following are true or false:

(a) If $\mathrm{Cov}[X, Y] = 0$ then X and Y are independent.

(b) If A and B are independent events, then $\Pr\{A \cup B\} = \Pr\{A\} + \Pr\{B\}$.

(c) $\mathrm{E}[aX + Y] = a\mathrm{E}[X] + \mathrm{E}[Y]$ assuming they are well-defined.

(d) $\mathrm{Var}[aX + Y] = a^2\mathrm{Var}[X] + \mathrm{Var}[Y]$ assuming they are well-defined.

(e) X is a Poisson random variable with parameter λ, then the variance of X is λ^2.

[30] 4. Suppose X and Y have the following joint probability mass function $\Pr\{X = i, Y = j\} = cij^2$ for positive integers i and j less than or equal to 2. Compute the following:

(a) Find the marginal probability mass function of X.

(b) Find the conditional probability mass function of X given $Y = 2$.

(c) Are X and Y independent? Explain.

(d) What is $\mathrm{Cov}[X, Y]$?

(e) Compute $\Pr\{X + Y \text{ is odd}\}$.

5. Suppose we have an AS/RS with a rectangular storage rack that is 20 meters long, but only 10 meters high. The retrieval device has two motors: one that moves the mast up and down the aisle at speed 2 meters per second, and the other raises and lowers device which removes the tray from the rack at speed 2 meters per second. Assume that the lower left corner of the rack is designated (0,0) and that a tray must be retrieved from a random location (X, Y). Assume that 50% of the items cause 90% of the activity, and that these active items have been stored in the left
[30] half of the rack.

   (a) Find the joint probability density function of (X, Y).

   (b) Let T be the travel time from the origin to the random location (X, Y). Note that $T = \max\{X/2, Y/2\}$. Determine the probability that it takes less than 3 seconds to get to the desired location; i.e., compute $\Pr\{T < 3\}$.

   (c) Set-up an integral for the expected travel time using the joint pdf that you determined in part a, but do not do the integration.