Name: ____________________

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ISyE 3027
Test 3

[25] 1. Suppose we are manufacturing chairs. Make a reasonable guess for the distribution of the random variable X where X is:

(a) the number of flaws on the finish of the first chair,

(b) the number of chairs that do not have to be repaired among the first 20,

(c) the number of chairs produced until a chair that needs to be repaired is produced,

(d) the total weight of glue used among the first 1000 chairs,

(e) the length of time until a chair is dropped by a worker.

[25] 2. Suppose X is a random variable with mean 3 and variance 16, and Y is a random variable with mean 5 and variance 4.

(a) What is the mean of $3Y - 5$?

(b) What is the variance of $3Y - 5$?

(c) What is the mean of $X - 3Y + 4$?

(d) What is the variance of $X - 3Y + 4$ assuming X and Y are independent random variables?

(e) What is the variance of $X + Y$ assuming that $Cov(X, Y) = 1$?

[25] 3. Let Y be a random variable with probability density function $f(t) = ct$ for $0 \leq t \leq 2$. Compute the following:

(a) c

(b) $\Pr\{Y \geq 1\}$

(c) $\Pr\{Y > 3/2 \mid Y > 1\}$

(d) $\mathrm{E}[Y]$

(e) $\mathrm{Var}[Y]$

[25] 4. Suppose X and Y are non-negative random variables with joint probability mass function $\Pr\{X = i, Y = j\} = cij$ for $1 \leq i + j \leq 4$. Compute the following:

(a) c

(b) the probability mass function of X

(c) the conditional probability mass function of X given $Y = 1$.

(d) the expected value of X conditioned on $Y = 1$.

(e) $\mathrm{E}[XY]$

(f) $\Pr\{X \leq 2, Y \leq 1\}$

(g) $\Pr\{X + Y \leq 2\}$

(h) $\Pr\{X = Y\}$

[40] 5. Suppose we have a building with a floor shaped like an isoceles right triangle. The two sides adjacent to the right triangle have length 100 feet. Think of the right angle being at the origin, and other two corners at $(100, 0)$ and $(0, 100)$. The overhead crane is located at the origin and needs to travel to a point (X, Y) which is uniformly distributed over the region. The crane has two motors one that moves the crane north and south, and the other that moves the crane east and west. Both motors move at the speed of 20 feet per minute. Since the motors can work at the same time, it is reasonable to assume that the length of time to go from the origin to (X, Y) is the maximum of two times: the time to go east from 0 to X, and the time to go north from 0 to Y. Let T be the length of time that it takes the crane to move from the origin to (X, Y).

(a) Can you determine the joint probability density function of (X, Y)? (If so, do so.)

(b) Find the marginal probability density function of X.

(c) Use symmetry to determine the marginal pdf of Y without integrating.

(d) Are X and Y independent? Explain.

(e) Derive an expression for the round trip time T as a function of X, Y, and the speed of the motors.

(f) Find $\Pr\{T > 2.5 \text{ minutes}\}$.

(g) Write down the double integral for $E[T]$ but do not integrate it.

(h) Do you think X and Y are positively correlated, negatively correlated, or uncorrelated? Why?

[10] 6. Suppose that $X_1, X_2, \ldots$ are i.i.d. random variables. The random variable X_i represents the time of the ith round trip from the previous problem. Assume that X_1 has mean time $\mu = 5$ minutes and variance $\sigma^2 = 9$ minutes^2. Use the central limit theorem, to estimate the probability that the total travel time for the first 16 trips is between 68 minutes and 92 minutes.