Name: ______________________________

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ISyE 3027
Test 3

1. Make a reasonable guess for the what the distribution of the random
[25] variable X might be where X is:
   (a) the number of fish caught during the first two hours of fishing,
   (b) the number of fish caught until a rainbow trout is caught,
   (c) the length of time until the first trout is caught,
   (d) the number of rainbow trout among the first 3 fish caught,
   (e) how many minutes since the top of the hour when the first fish is caught.

2. Suppose X is a random variable with mean 5 and variance 9, and Y is a
[25] random variable with mean 7 and variance 4.
   (a) What is the mean of $3Y - 5$?
   (b) What is the variance of $3Y - 5$?
   (c) What is the mean of $X - 3Y + 4$?
   (d) What is the variance of $X - 3Y + 4$ assuming X and Y are independent random variables?
   (e) What is the variance of $X + Y$ assuming that $Cov(X, Y) = 3$?

3. Let Y be a random variable with probability density function $f(t) = ce^{-}\mu t$
[25] for $t > 0$. Compute the following:
   (a) c
   (b) $\Pr\{Y \leq 1\}$
   (c) $\Pr\{Y > 3 \mid Y > 2\}$
   (d) $\mathrm{E}[Y]$
   (e) $\mathrm{Var}[Y]$

4. Suppose X and Y are non-negative random variables with joint probability mass function $\Pr\{X = i, Y = j\} = ci^2 j$ for $1 \leq i + j < 4$. Compute the
[25] following:
   (a) c
   (b) the probability mass function of X
   (c) $\mathrm{E}[XY]$

(d) $\Pr\{X \leq 2, Y < 1\}$

(e) $\Pr\{X + Y < 2\}$

[25] 5. Let X and Y have joint probability density function $f_{X,Y}(s,t) = 6e^{-(2s+3t)}$, $s > 0, t > 0$. Compute the following.

(a) the probability density function of X

(b) $\Pr\{\min(X,Y) > r\}$, $r > 0$.

(c) $\Pr\{X = Y\}$

(d) $\Pr\{X + Y < 2\}$

(e) Are X and Y independent?

[20] 6. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with mean μ and variance σ^2. Let Y be the average of the $X_1, \ldots, X_n$. That is,

$$Y = \frac{X_1 + \cdots + X_n}{n}$$

(a) Determine the $E[Y]$ as a function of n, μ, and σ^2.

(b) Same as (a) except for $\mathrm{Var}[Y]$.

[1] 7. Bonus: Whose needle?