Name: _____

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ISyE 3027
Test 3

1. [25] Make a reasonable guess for the what the distribution of the random variable $X$ might be where $X$ is:

   (a) the length of wall paper produced until there is a defect in the wallpaper,

   (b) the number of defects in the next roll of wallpaper produced,

   (c) the number of rolls that pass inspection out of the next 25 rolls,

   (d) the number of rolls produced until a roll fails to pass inspection,

   (e) the total weight of the next 100 rolls of wallpaper.

2. [30] Let $Y$ be a random variable with probability density function $f(t) = 3e^{-3t}$ for $t > 0$. Compute the following:

   (a) $\Pr\{Y = 2\}$

   (b) $\Pr\{Y \geq 2\}$

   (c) $\Pr\{Y > 3 \mid Y > 2\}$

   (d) $\mathrm{E}[e^{tY}]$

   (e) $\Pr\{Y > 2 \mid Y > 3\}$

   (f) What's the name of this distribution?

3. [30] Suppose $X$ and $Y$ are non-negative random variables with joint probability mass function $\Pr\{X = i, Y = j\} = c(i+j)$ for non-negative integers $i$ and $j$ with $i + j \leq 3$. Compute the following:

   (a) $c$

   (b) the probability mass function of $X$

   (c) $\mathrm{E}[XY]$

   (d) $\Pr\{X = Y\}$

   (e) $\Pr\{X + Y < 3\}$

   (f) Are $X$ and $Y$ independent? Explain.

4. [30] Let $X$ and $Y$ have joint probability density function $f_{X,Y}(s,t) = cs, 0 < s < 1, 0 < t < 2$. Compute the following.

   (a) $c$.

(b) the marginal probability density function of $Y$.

(c) the conditional probability density function of $X$ given $Y = 1$.

(d) $\Pr\{X = Y\}$

(e) $\Pr\{X + Y < 1\}$

(f) Are X and Y independent? Explain.

[20]

5. Suppose you have 3 items remaining in stock to be sold. For each item you sell, you receive a \$100 bonus. You cannot sell more than three. Suppose the demand for these items tomorrow is a Poisson random variable with a mean of 2. Find the expected total bonus that you will receive tomorrow.