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ISyE 3027  
Test 3

- [30] 1. Make a reasonable guess for the distribution of each of the following:
- (a) the number of cars that go by until there is a car carrying 3 people,
  - (b) the number of cars among the next 20 cars that have more than two bumper stickers,
  - (c) whether the next vehicle contains a child or not,
  - (d) the number of cars in the next 10 minutes containing two or more children,
  - (e) the length of time until the next ambulance,
  - (f) the total number of people in the next 50 cars.
- [20] 2. Suppose we are dealt 7 cards from a well-shuffled standard deck.
- (a) What is the probability of two triples, i.e., 3 of one rank, 3 of another rank, and 1 card of a 3rd rank?
  - (b) Define a full yacht to be 4 of one rank, and 3 of another. What is the probability of a full yacht?
- [20] 3. Determine whether each of the following are true or false and explain:
- (a)  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$ ,
  - (b)  $\Pr(B \cap A) = \Pr(B | A) \Pr(A)$ ,
  - (c) If the covariance between  $X$  and  $Y$  is zero, then  $X$  and  $Y$  are independent.
  - (d) Let  $X$  and  $Y$  be independent, exponentially distributed random variables with parameters  $\lambda$  and  $\mu$ , respectively. The  $\min(X, Y)$  is exponentially distributed with a parameter  $\lambda + \mu$ .
- [20] 4. Suppose  $\Pr\{X = i\} = (2 + i)/10$  for  $i = -1, 0, 1, 2$  and zero otherwise.
- (a) Compute  $\Pr\{X \leq 3/2\}$ .
  - (b) Compute  $\Pr\{X = 0 | X \leq 1\}$ .
  - (c) Compute the mean of  $X$ .
  - (d) Compute the variance of  $X$ .

[20]

5. Let  $Y$  be a random variable with probability density function  $f(t) = (3/2)t^2$  for  $-1 \leq t \leq 1$ . Compute the following:

- (a)  $\Pr\{Y \geq 1/2\}$
- (b)  $\Pr\{Y > 1/2 \mid Y > 0\}$
- (c) The mean of  $Y$
- (d) The variance of  $Y$

[20]

6. Suppose we have items stored in a carousel. The carousel is bidirectional which means that it can rotate in either direction. The time to complete an entire rotation in one direction is 24 seconds. Of course, when a request arrives, the carousel is rotated in the direction that yields the shorter rotation time.

- (a) What is a reasonable assumption for the distribution of the rotation time to the pick location?
- (b) What is the probability density function for this distribution?
- (c) What is the mean rotation time to a pick location?
- (d) What is the variance of the rotation time to a pick location?

[20]

7. Suppose the pick times are exponentially distributed with a mean of 2 seconds.

- (a) What is the probability density function for this distribution?
- (b) What is the variance of the pick time?
- (c) What is the expected remaining pick time if the picker has been working for 1.5 minutes and has not finished?
- (d) What is the median of the pick time distribution?

[20]

8. (This problem is a continuation of the previous two problems.) Suppose 100 items need to be picked. Assume that the locations of the items are independent, identically distributed random variables, and that items are picked in the order given. Thus, to pick an item, the carousel first rotates to the desired location, and then the item is picked. Let  $T$  be the total time to retrieve the 100 items.

- (a) What is the mean of  $T$ ?
- (b) What is the variance of  $T$ ?
- (c) What is the probability that all 100 items are retrieved within 15 minutes?
- (d) What would be a better way to retrieve the items?