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ISyE 3027

Test 2

[30]

1. Make a reasonable guess for the what the distribution of the random variable X might be where X is assuming that you are watching cars in the HOV lane underneath the North Avenue bridge.

   (a) the number of cars that go by until a car containing only one person,

   (b) the number of cars among the next 15 cars that have more than three people,

   (c) whether the next vehicle is a motor cycle or not,

   (d) the number of motorcycles in the next 10 minutes,

   (e) the length of time until the next motorcycle,

   (f) a vector whose first component is the number of cars, the second component is the number of trucks, and the third component is the number of motorcycles out of the next 15 vehicles. Assume that there are no other vehicles besides these three.

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2. Suppose $\Pr\{X = i\} = c(i + 1)$ for $i = 0, 1, 2$ and zero otherwise.

   (a) Determine c.

   (b) Compute $\Pr\{X \leq 3/2\}$.

   (c) Compute $\Pr\{X = 0 | X = 2\}$.

   (d) Compute the mean of X.

   (e) Compute the variance of X.

   (f) Suppose a bonus is received if $X = 2$ of 1000 dollars. What is the expected bonus?

[30]

3. Let Y be a random variable with probability density function $f(t) = 2 - 2t$ for $0 \leq t \leq 1$. Compute the following:

   (a) $\Pr\{Y = 1/2\}$

   (b) $\Pr\{Y \geq 1/2\}$

   (c) $\Pr\{Y > 3/4 \mid Y > 1/2\}$

   (d) The mean of Y

   (e) $E[Y^2]$

   (f) The variance of Y

4. Suppose we have two dice, and that each die has 4 sides labelled 1 through 4. Suppose one die is red and the other blue. Roll the dice, and let X be the number that shows up on the red die, and let Y *be the total of the two dice.* Compute the following:

[30]

   (a) Find the joint probability mass function of X and Y

   (b) Find the marginal probability mass function of X

   (c) Find the conditional probability mass function of X given $Y = 4$.

   (d) Find the expected value of X conditioned on $Y = 4$.

   (e) Suppose you win if the red die is a one, and in this case you receive Y dollars. What is the average amount you receive?

   (f) Are X and Y independent? Explain.

5. Suppose we have an AS/RS with a square storage rack that is 20 meters long. The retrieval device has two motors: one that moves the mast up and down the aisle, and the other raises and lowers device which removes the tray from the rack. Assume that the lower left corner of the rack is designated (0,0) and that a tray must be retrieved from a random location (X, Y). Assume that 25% of the items cause 80% of the activity, and that these active items have been stored in the lower left corner of the rack.

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   (a) Find the joint probability density function of (X, Y).

   (b) Assuming that each motor causes the retrieval device to move at 2 meters per second either horizontally or vertically, and that the travel time is the maximum of the horizonal travel time and the vertical travel time. Let T be the travel time from the origin to (X, Y). Find the cumulative distribution function of T.

   (c) Are X and Y independent? Explain.