Recap

- Last class (October 18, 2016)
  - Repeated games where each stage has a sequential game
    - Wage-setting
  - Games of incomplete information
    - Cournot competition with incomplete information
    - Battle of the sexes where payoffs are private information

- Today (October 24, 2016)
  - Principal-agent models

Principal-agent examples

- Restaurant owner - waiter
- Software company - salesman
- Auto manufacturer - customer leasing a car
- Insurance company - insured
- Donor - NGO
- Global NGO - Local organization delivering goods/services
Example

- The principal offers wage $w$
- If the agent accepts the offer
  - Agent can put “high” ($e=25$) or “low” ($e=0$) effort
  - Agent’s utility: $U(w,e) = w - e$
- Agent’s reservation level of utility: 81
- Principal’s payoff
  - $270$, if the agent works hard
  - $70$, if the agent doesn’t work hard

First-best contract

- The agent won’t accept the job, unless the wage exceeds his reservation utility:
  - $w \geq 81$
- Employing this agent is worthwhile to the principal only if the agent works hard (otherwise, the principal only gets 70)
- For the agent to work hard, his utility from working hard should exceed his reservation utility:
  - $U(w,e) \geq 81$
  - $w - 25 \geq 81 \rightarrow w \geq 106$
- First-best contract: offer $106 + \epsilon$ to the agent and “trust” that he will work hard
Moral hazard

- First-best contract: Offer the agent $106 + \varepsilon$
- What is the problem with this contract?

“Moral hazard”: the agent takes a decision or action that affects his or her utility as well as the principal’s, the principal only observes the “outcome” (as an imperfect signal of the action taken), and the agent does not necessarily choose the action in the interest of the principal.

Alternative: Offer a contract where the wage depends on the effort level.

Contract conditioned on effort level

- Offer two wage rates:
  - $w^H$ if the agent exerts high effort
  - $w^L$ if the agent exerts low effort
- How to choose $w^H$ so that accepting the offer and working hard is desirable for the agent?
  - $w^H - 25 \geq 81$ participation constraint
  - $w^H \geq 106$ (individual rationality constraint)
  - $w^H - 25 \geq w^L$ incentive constraint
- What is the problem with this contract?
  - Difficult to enforce
Contract conditioned on outcome under uncertainty

- Suppose the agent is a salesman representing the principal to a client
- Three possible outcomes
  - The client places no order ($0)
  - The client places a “small” order ($100)
  - The client places a “large” order ($400)
- Probabilities for different outcomes under each effort level

<table>
<thead>
<tr>
<th>Effort</th>
<th>No order ($0)</th>
<th>Small order ($100)</th>
<th>Large order ($400)</th>
<th>Expected order size</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
<td>$270</td>
</tr>
<tr>
<td>Low</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>$70</td>
</tr>
</tbody>
</table>

\((0.1)(0) + (0.3)(100) + (0.6)(400) = $270\)

Contract conditioned on outcome

- A contract where the wage depends on the observable outcome
  - No order → pay the agent \(x_1\)
  - Small order → pay the agent \(x_2\)
  - Large order → pay the agent \(x_3\)
Contract conditioned on outcome

If the worker works hard:

Principal’s expected profit =

$$(0.1)(0-x_1) + (0.3)(100-x_2) + (0.6)(400-x_3) = K$$

no order  small order  large order

→ Consider a contract with $$x_1$$, $$x_2$$, $$x_3$$, such that the principal’s profit is the same regardless of the outcome!

$$(0.1)(0-x_1) + (0.3)(100-x_2) + (0.6)(400-x_3) = (0.1)K + (0.3)K + (0.6)K = K$$

Contract conditioned on outcome

Expected profit =

$$(0.1)(0-x_1) + (0.3)(100-x_2) + (0.6)(400-x_3) = (0.1)K + (0.3)K + (0.6)K = $K$$

$$x_1 = -K$$

$$x_2 = 100-K \iff 100-x_2=K$$

$$x_3 = 400-K \iff 400-x_3=K$$

Expected wage:

$$(0.1)x_1 + (0.3)x_2 + (0.6)x_3 = (0.1)(-K) + (0.3)(100-K) + (0.6)(400-K)= -K+270 \geq 106 \implies K \leq 164$$

→ $K \leq 164$
Contract conditioned on outcome

If we set $K = 164$

$x_1 = -164$
$x_2 = 100 - K = -64$
$x_3 = 400 - K = 236$

- A contract where wage depends on the observable outcome
  - No order $\rightarrow$ agent pays the principal $164$
  - Small order $\rightarrow$ agent pays the principal $64$
  - Large order $\rightarrow$ principal pays agent $236$

Principal's (expected) revenue if the agent works hard: $270$  Expected profit: $164$

How much does the principal's revenue differ from the expected revenue under each outcome?

- No order $\rightarrow$ 0-270 = -270 -270 + 106 = -$164$
- Small order $\rightarrow$ 100-270 = -170 -170 + 106 = -$64$
- Large order $\rightarrow$ 400-270 = 130 130 + 106 = $236$

Principal's profit

- No order $\rightarrow$ $164$
- Small order $\rightarrow$ $164$
- Large order $\rightarrow$ $164$
Contract conditioned on outcome

- Agent's choices and (expected) payoffs under each choice (assuming the agent is risk-neutral)
  - Reject the contract and get reservation utility $81
  - Accept the contract and don't work hard
    \[(0.1)(236)+(0.3)(-64)+(0.6)(-164)-0= -94\]
  - Accept the contract and work hard
    \[(0.6)(236)+(0.3)(-64)+(0.1)(-164)-25= 81\]

The principal designed the contract such that the agent internalizes the effect of his effort decision and bears fully the cost of putting low effort.

<table>
<thead>
<tr>
<th>Wages</th>
<th>No order ((-164))</th>
<th>Small order ((-64))</th>
<th>Large order ((236))</th>
</tr>
</thead>
<tbody>
<tr>
<td>High effort</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Low effort</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Contract with positive wages

- Suppose the agent only accepts positive wages.
- What are the wages \(x_1, x_2\) and \(x_3\) corresponding to no order, small order and large order outcomes that maximize the principal's payoff?
- Participation constraint
  \[0.6 \ x_3 + 0.3 \ x_2 + 0.1 \ x_1 - 25 \geq 81\]
- Incentive constraint (return from work \(\geq\) return from shirk)
  \[0.6 \ x_3 + 0.3 \ x_2 + 0.1 \ x_1 - 25 \geq 0.6 \ x_1 + 0.3 \ x_2 + 0.1 \ x_3\]
- Nonnegativity constraint: \(x_1, x_2, x_3 \geq 0\)
- Principal's objective
  Maximize \(0.6(400- x_3)+0.3(100- x_2)+0.1(0- x_1)\).
  Equivalently, Minimize \(0.6 \ x_3 + 0.3 \ x_2 + 0.1 \ x_1 \)

Many solutions to the LP: e.g., 118, 117, 1
Risk aversion

- What if the agent is risk-averse?
  - A person who prefers to get the expected value of a gamble for sure instead of taking the risky gamble is risk averse
  - E.g.: getting $25 for sure vs. getting $0 with probability 0.75 and $100 with probability 0.25
  - The agent and the principal may have different “beliefs” about the probabilities of different outcomes under different effort levels

Example – Risk averse agent

- Agent’s reservation utility = 10
- Agent’s possible actions if accepts the contract: work hard (e=2), don’t work hard (e=0)
- Two possible outcomes: L and H
- Principal offers wages $w^L$ and $w^H$ based on the outcome
Example – Risk averse agent

- Probabilities of H and L outcomes
  - Agent does not work hard
    - H with probability 0.4
    - L with probability 0.6
  - Agent works hard
    - Principal’s belief
      - H with probability 0.8
      - L with probability 0.2
    - Agent’s belief
      - H with probability 0.7
      - L with probability 0.3