Dynamic Pricing in the Presence of Inventory Considerations: Research Overview, Current Practices and Future Directions[1]

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Abstract

The benefits of dynamic pricing methods have long been known in industries, such as airlines, hotels and electric utilities, where the capacity is fixed in the short-term and perishable. In recent years, there has been an increasing adoption of dynamic pricing policies in retail and other industries as well, where the sellers have the ability to store inventory. Three factors contributed to this phenomenon: the increased availability of demand data, the ease of changing prices due to new techologies, and the availability of decision-support tools for analyzing demand data and for dynamic pricing. This paper constitutes a review of the literature and current practices in dynamic pricing. Given its applicability in most markets and its increasing adoption in practice, our focus is on dynamic (intertemporal) pricing in the presence of inventory considerations.
(*Dynamic Pricing; E-commerce; Revenue Management; Inventory*)

1   Introduction

In recent years, we have witnessed an increased adoption of existing dynamic pricing strategies and their further development in retail and other industries [18]. Three factors contributed to this phenomenon: the increased availability of demand data, the ease of changing prices due to new techologies, and the availability of decision-support tools for analyzing demand data and for dynamic pricing. Determining the "right" price to charge a customer for a product is a complex task, requiring that a company know not only its own operating costs and availability of supply but also how much the current customer values the product and what future demand will be. Therefore, in order to charge a customer the right price, a company must have a wealth of information about its customer base and be able to set and adjust its prices at minimal cost. Until very recently, neither element was present; companies had limited ability to track information about their customers' tastes, and faced high costs in changing prices. Today, in both Internet and brick-and-mortar stores, new

technology allows retailers to collect information not only about the sales, but also about demographic data and customer preferences. Due to the ease of making price changes on the Internet, dynamic pricing strategies, especially in the form of price markdowns, are now frequently used in B2C as well as B2B commerce by numerous companies, including FairMarket, Comp USA, Lands' End, J.C. Penney, MSN Auction and Grainger. Although price changes are still costly in traditional retail stores, this may soon change with the introduction of new technologies such as Electronic Shelf Labeling Systems [69]. These advances in technology have opened the door for dynamic pricing optimization solutions providers (DPOSP), who offer decision-support tools for sophisticated pricing strategies utilizing a blend of complex optimization methods. Even with limited ability to change prices today, early users of the new pricing decision-support software have reported improved financial performance, quick return on investment, and no negative impact on price image. AMR Research predicts that the market for pricing optimization tools should grow from about $75M in 2002 to at least $500M by 2005 [30].

While the types of pricing policies/methods used in the exchange of goods and services vary greatly, they fall into two broad categories: *posted price* mechanisms and *price discovery* mechanisms. Under a posted price mechanism, a good is sold at a take-it-or-leave-it price determined by the seller. In price discovery mechanisms, prices are determined via a bidding process, such as auctions. Our focus in this paper is on dynamic posted price mechanisms (we refer the reader to [42] and [50] for a comprehensive review of the auction literature).

In the past, companies would fix the price of a product or service over a relatively long time period, i.e., the posted prices were usually *static*. This was mainly due to the absence of accurate demand information, the high transaction costs associated with changing prices and the huge investments required for software and hardware necessary for implementing a dynamic pricing strategy. *Dynamic* posted prices are also take-it-or-leave-it prices, where the seller changes prices dynamically over time (intertemporal prices), based on factors such as the time of sale, demand information, and supply availability. With the goal of balancing demand and supply, early applications of dynamic pricing methods have been mainly in industries where the short term capacity (supply) is difficult to change, such as airlines, cruise ships, hotels, electric utilities, sporting events, and health care [28] [29] [51] [81][2]. In most of these industries it was possible to control prices in a centralized fashion and prices could be changed at little or no cost. In contrast, in industries such as retail where short term supply is more flexible (e.g., through inventory replenishment) or where price changes

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[2]Dynamic pricing practices in these industries are known as yield management (or revenue management, or perishable asset revenue management). The key factor behind revenue management is the segmentation of the market into multiple classes (e.g., business vs. leisure customers), where different types of 'products' (e.g., fully refundable fares vs. restricted fares that require Saturday night stays) are targeted to each class (second degree price discrimination). The main focus in revenue management research has been on the allocation of limited capacity to different demand classes. For example, in the airline industry, given a booking request that asks for seats in an itinerary in a specific booking class, the fundamental revenue management decision is whether to accept or reject this booking, i.e., overbooking and seat inventory control [51]. In this domain, most of the literature on pricing decisions focuses on industry-level trends and implications, rather than operational level decisions. Recent papers that discuss revenue management research and practices include [3] [41] [51] [81].

are costly, the focus has been on improving inventory management practices. Advances in information technology and e-commerce have played a significant role in improved inventory management. For example, programs such as CPFR (collaborative planning, forecasting and replenishment), QR (quick response) and VMI (Vendor Managed Inventory) enable information sharing and collaboration among supply chain partners, lowering inventory costs while simultaneously increasing service levels. However, despite significant improvements in reducing supply chain costs via improved inventory management, a large portion of retailers still lose millions of dollars annually due to lost-sales and excess inventory. Therefore, many are now willing to look at the demand side of the supply-demand equation, re-examine their pricing policies and explore dynamic pricing software technologies for better demand management.

This paper constitutes a review of current practices in dynamic pricing as well as the dynamic pricing literature. To better understand the state-of-the-art pricing practices, we conducted interviews with the Directors of Marketing and Operations of leading DPOSPs in the field, including DemandTec, Khimetrics, ProfitLogic and Spotlight Solutions. We also spoke with Manugistics, i2 and Retek, leaders in supply-chain management and ERP software, who are moving into price optimization solutions. Based on our discussions with DPOSPs and the dynamic pricing literature from the fields of operations research, economics and marketing, we suggest a categorization of market environments for which distinct dynamic pricing problems arise (Section 2). We then discuss the price optimization solutions currently offered by DPOSPs, the challenges they face in their implementation, provide an overview of the existing relevant literature, and propose future directions for research (Sections 3 and 4). While most of the literature reviewed here is motivated by a physical retail or purchasing environment, the results and insights are applicable both to brick-and-mortar and online selling environments. There are, however, some unique characteristics of the online selling environment, which offer additional flexibilities and challenges in pricing decisions. We conclude by discussing the impact of these unique characteristics of online stores on pricing decisions and additional research directions in Section 5.

Given its applicability in most markets and its increasing adoption in practice, our focus in this survey is on dynamic (intertemporal) pricing in the presence of inventory considerations. A short list of references and discussion on other branches of the pricing literature not reviewed here can be found in the online Appendix. Our survey complements other surveys of the pricing literature from the operations research/management science community that focus on revenue management for fixed, perishable capacity [6] [51], and coordinated pricing and production/procurement decisions [12] [82], from the marketing community that focus on how markets behave [55], goods with dependent demands (Bass models) [62] [63] [54] and brand loyalty and switching [26], and finally from the economics community that focus on price discrimination [73] [76]. We complement these surveys by focusing on dynamic pricing in the presence of inventory considerations, providing a critical analysis of the current practices in dynamic pricing, highlighting the potential for dynamic pricing in ecommerce, and presenting a substantial list of future research directions.

2   Dynamic Pricing Across Markets: A Categorization of the Literature

One can easily imagine that the dynamic pricing problem facing a retailer of a durable good, such as a refrigerator, at the start of its life cycle is unlike that facing a retailer selling bathing suits at the end of summer. While both retailers must decide how to change prices over the remaining selling horizon, the factors that affect their decisions are very different. We postulate that there are three main characteristics of a *market environment* that influence the type of dynamic pricing problem a retailer faces:

(1) **Replenishment vs. No Replenishment of Inventory (R/NR)**: Whether or not inventory replenishment is possible during a price planning horizon affects whether a seller needs to make inventory decisions upfront, before the selling season starts, or whether she will have access to extra units during the selling season. For some short life cycle products, such as holiday paraphernalia and certain fashion apparel, replenishing inventory during the selling season is usually not possible due to long procurement lead times. In such cases, the retailer needs to make pricing decisions given a fixed amount of inventory. For other products, the retailer is able to replenish her inventory periodically.

(2) **Dependent vs. Independent Demand Over Time (D/I)**: Demand of a product across multiple periods can be dependent if the product is a durable good or if customers' knowledge about the product plays a role in their decision to make a purchase. For a durable good, by definition, the life of the product is longer than the time horizon over which the retailer makes price changes. One can model the size of the total demand pool for a durable good as being fixed (although it may be unknown), with no (or limited) repeat purchases over the relevant selling horizon. In such an environment, a sale today is one less possible sale tomorrow. Alternatively, customers' knowledge about a product can affect their decision to purchase and their willingness to pay. In general, customers are less informed about a product at the start of its life cycle and a retailer can influence future demand via word-of-mouth from previous sales and advertisement effects. In contrast, for most non-durable goods demand is independent over time, i.e., current sales do not negatively impact future sales. This is certainly true for most necessity items, such as milk and bread, where consumers make frequent repeat purchases. For seasonal or fashion goods, the selling horizon is usually too short to allow for any significant knowledge acquisition by customers to impact the demand.

(3) **Myopic vs. Strategic Customers (M/S)**: The purchasing behavior of the customers affects the seller's pricing decisions over time. A myopic customer is one who makes a purchase immediately if the price is below his valuation (reservation price), without considering future prices. Myopic (or non-strategic) customer behavior allows the seller to ignore any detrimental effects of future price cuts on current customer purchases. Conversely, a strategic (or rational) customer takes into account the future path of prices when making purchasing decisions. Dynamic pricing decisions of a seller facing strategic customers is more complex, since the seller has to consider the effects of future as well as current prices on customers' purchasing decisions.

In addition to the three main characteristics discussed above, numerous other factors can influence a dynamic pricing policy, such as business rules, cost of implementing price changes, seasonality of and external shocks to demand, and cross-elasticities. We briefly discuss some of these additional characteristics in the online Appendix.

In our review of the relevant dynamic pricing literature, we found that the bulk of the existing pricing-with-inventory literature can be partitioned into two categories based on the following market environments: NR-I (NR-I-M and NR-I-S) and R-I-M. The first category focuses on market environments where there is no opportunity for inventory replenishment over the selling horizon and demand is independent over time (NR-I, Section 3). NR-I markets arise when the seller faces a short selling horizon, e.g., when the product itself is a short-life cycle product, such as fashion apparel or holiday products, or is at the end of its life-cycle (e.g., clearance items). In these markets, production/delivery lead times prevent replenishment of inventory and hence, the seller has a fixed inventory on hand and must decide on how to price the product over the remaining (short) selling horizon. Depending on whether customers behave myopically or strategically, these markets and the relevant literature can further be divided into two subcategories, NR-I-M and NR-I-S.

The second category focuses on market environments where the seller may replenish inventory over time, demand is independent over time, and customers behave myopically (R-I-M, Section 4). In these markets, customers purchase a product without regard to future price paths, and may make frequent purchases over the selling horizon (non-durable good). Modeling customer behavior as myopic can be appropriate in several settings; (i) the items sold in these markets are necessity items and customers cannot afford to wait for a price drop; (ii) the prices or price changes are small enough such that strategic waiting for lower prices does not provide much value; (iii) the customer pool is sufficiently large such that any one customer's purchase has no effect on prices; and (iv) impulse purchases. This category accurately captures most non-durable products, such as grocery items, produce, and pharmaceutical products. R-I-M markets can be interpreted as having a medium-length selling horizon, where the seller makes inventory replenishment as well as pricing decisions in the presence of production/ordering and holding costs and possible lags in supply availability.

The focus of the academic literature on these two market settings coincides with the current offerings of the leading DPOSPs, who have targeted their products primarily to NR-I-M and/or R-I-M markets. Faced with a possibility of unprofitable inventory at the end of a product's life cycle, sellers of fashion and specialty items (NR-I markets) were the first to turn to DPOSPs to develop pricing software. Soon afterwards, retailers in grocery and drug stores (R-I markets) approached DPOSPs to help them design decision-support tools for the pricing of (non-durable) consumer package goods. The products in these markets are typically sold at very low margins and retailers felt that there were significant potential benefits to be gained from exploiting dynamic pricing opportunities or coordinating inventory and pricing decisions.

Given its applicability in most markets and its increasing adoption in practice, our focus in this survey is on dynamic (intertemporal) pricing in the presence of inventory considerations, i.e., dynamic pricing in NR-I (NR-I-M and NR-I-S) and R-I-M markets.

3 Market Type NR-I: No Inventory Replenishment and Independent Demand Over Time

In the last couple of decades, the variety of goods sold in the market has increased significantly, while the product life cycles have become shorter. Although improved supply chain practices and production technologies helped increase responsiveness, long lead times due to overseas imports and shorter selling seasons result in larger forecasting errors and an inability to change inventory levels in response to demand. As a result, production/inventory decisions have to be made in advance with little information about demand, before the actual selling begins. Given that the inventory levels and the length of the selling season are predetermined, pricing decisions become increasingly important in balancing demand and supply. In addition to short life cycle items (or items with a short life cycle relative to their production leadtime), fixed inventories with no replenishment are also the case in practice for items that are at the end of their life cycle. We discuss the decision-support tools for markdown pricing in Section 3.2 following the academic literature on pricing policies for NR-I markets in Section 3.1. We point out the missing links between the practical implementations, academic literature, and the current needs for decision-support tools in Section 3.3.

3.1 Literature Review

The percentage of markups and the frequency of sales have increased significantly in the last couple of decades [52]. Empirical (and some analytical) studies try to explain these phenomena using different hypotheses, including the fashion hypothesis [46] [56] [57], peak-load hypothesis and the thick market hypothesis [80]. We refer the reader to the online Appendix for this line of research, which focuses on 'why' prices follow a particular pattern over time.

Analytical models that study 'how' pricing decisions should be made in NR-I markets are presented in [7] [8] [24] [28] [46] [67] [84]. Common assumptions in these papers are: (1) the firm operates in a market with imperfect competition (e.g., a monopolist); (2) the selling horizon T is finite (a fixed sell-thru date); (3) the firm has a finite stock of n items and no replenishment option during the selling horizon; (4) investment made in inventory is sunk cost; (5) demand decreases in price (p); and (6) unsold items have a salvage value. These assumptions fit well to a large segment of retailing, where the production/ordering times of seasonal and fashion goods range from three to six months, compared to a short selling season of less than 12 weeks. Similarly, when selling the excess inventory of a product that is at the end of its life cycle, the inventory as well as the selling time is fixed. Hence, given a limited inventory without the option of replenishment, the goal is to price the products to maximize expected (discounted) profits over the (short) selling season. These assumptions, except possibly (1), are also common to most of the currently available markdown pricing decision-support tools.

One of the most important elements that influences pricing decisions is demand, and how it

reacts to price changes and other factors. In particular, pricing decisions need to consider the arrival process of (potential) customers and the changes in the customers' willingness to pay (reservation prices, or valuations) over time. Elmaghraby et al. [21] study a deterministic model where all the customers are available at the beginning of the selling horizon with known valuations. In Lazear [46], N customers arrive in each period with a reservation price V, where N is known to the seller and V is unknown but is drawn from a known distribution. Gallego and Van Ryzin [28] and Feng and Gallego [24] model the demand as a homogenous (time-invariant) Poisson process with intensity $\lambda(p)$, where $\lambda(p)$ is non-increasing in p. By charging a price p_t at time t, the firm controls the intensity of the demand.

In the above papers, the reservation prices or their distribution remain constant over time. In contrast, Bitran et al. [7], Bitran and Mondschein [8], and Zhao and Zheng [84] generalize these models by modeling the demand as a nonhomogenous Poisson process with rate λ_t and allowing the probability distribution of the reservation price ($F_t(x)$) to change over time. One motivation for this model is that price markdowns are rarely advertised, therefore customers have little information about prices before they go to a store. Hence, in this model the arrival rate depends on time (not on price) and the purchase rate depends on the reservation price of customers. The demand process for a given pricing policy is a nonhomogenous Poisson process with intensity $\lambda(p, t) = \lambda_t(1 - F_t(p))$. In addition to price and time, Smith and Achabal [67] incorporate the impact of the inventory level on demand. Particularly in the retail and fashion goods industry, the demand of a product is usually influenced by the shelf space it occupies. The relationship between "display" area and sales is typically one-sided: too low an inventory level may slow the sales rate while inventory above the critical "minimum" level, f_0, does nothing to promote further sales. For this reason, retailers often define a minimum on-hand inventory for each product to ensure that the product receives adequate presentation. Smith and Achabal consider a deterministic continuous demand model where demand at time t is given by $x(p, I, t) = k(t)y(I)e^{-\gamma p}$, where $k(t)$ is the seasonal demand at time t, $y(I)$ is the inventory effect when inventory level is I and $e^{-\gamma p}$ is the sensitivity of demand to price p. All the papers reviewed in this section, except Elmaghraby et al. [21], assume myopic customers.

An interesting question is how the shape of the price path looks like in an NR-I market when prices are allowed to move in either direction over time. Depending on the underlying modeling assumptions, current research suggests that prices either decrease over time [46] or prices move both up and down [8] [24] [28]. In the latter case, there is sometimes a general downward trend [8] [28]. Two modeling assumptions that lead to the optimality of strictly decreasing prices in [46] are myopic customer behavior and equal customer valuations. Under these assumptions, decreasing prices facilitates demand discovery: if a product is not sold at price p, the seller can infer that the customers are willing to pay less than p and reduce the price [46]. Conversely, prices may increase when customers arrive stochastically over time, and where it is possible for customers who arrive later during the selling time horizon to have higher valuations for the product.

Lazear [46] and Elmaghraby et al. [21] study *periodic* pricing policies, where prices are

updated at fixed time intervals. In contrast, the other papers discussed in this section consider *continuous* pricing policies where (i) the price path can be a continuous function of time, or (ii) given a discrete set of allowable prices, the time between two price changes is a decision variable. Intuitively, in case of periodic pricing, a group of customers arrive over the length of a period and are offered the same price. In contrast, in case of continuous pricing, customers arrive sequentially over time, and as a result, each potential customer could be offered a different price.

Lazear [46] studies the pricing of a single good when all buyers have the same reservation price (valuation) drawn from a known distribution function. Using dynamic programming, he shows that having two periods (and prices) for selling the good increases expected profits, mainly because the seller can price the good at a higher price in the first period, and if the good is not sold, she can update her belief about the valuations and drop the price in the second period. The results are extended to the case where some of the customers are just "shoppers", with no intention of buying the good (i.e., shoppers have a valuation of zero for the product). A customer is a shopper with probability p and a real buyer with probability $1-p$, and these probabilities are known by the seller. As the number of shoppers (vs. real buyers) increases, the seller can infer very little about the valuations of customers, and hence the two-period problem becomes similar to having two independent one-period problems, and prices tend to be constant over time. Lazear finds that in the optimum, prices start high and then fall over time such that the probability of making a sale is equal in every time period. Extending the two-period results to T periods, Lazear shows that as T increases, the initial price increases, the final price goes to zero, and prices drop by smaller amounts. Lazear also considers the sales of multiple units; under the assumption of homogenous customers in each period, the results are very similar to the case of a single good. Pashigian [56] and Pashigian and Bowen [57] investigate the issues and the model presented in [46] empirically.

Gallego and Van Ryzin [28] model the continuous pricing problem using intensity control theory [10]. For a "regular" demand function[3], they derive optimality conditions and show that
(P.I) at a given point in time, the optimal price decreases as the inventory increases,
(P.II) for a given level of inventory, the optimal price rises if there is more time to sell, and
(P.III) more on hand inventory and/or a longer remaining selling horizon lead to higher expected revenues.
To find closed-form solutions, they look at the special case of $\lambda(p) = ae^{-\alpha p}$. They show that the price jumps up after each sale, then decays slowly until the next sale, and jumps up again. Bitran and Mondschein [8] and Zhao and Zheng [84] generalize the model of Gallego and Van Ryzin by modeling the demand as a nonhomogenous Poisson process with intensity $\lambda(p,t) = \lambda_t(1 - F_t(p))$, where $F_t(.)$ is the distribution of reservation prices. They show that (P.I) holds under this more general model as well; however, (P.II) may not hold

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[3]In a "regular" demand function there is a one-to-one correspondence between prices and demand rates, i.e., $\lambda(p)$ has an inverse denoted by $p(\lambda)$, and the revenue rate $r(\lambda) = \lambda p(\lambda)$ satisfies $\lim_{\lambda \to 0} r(\lambda) = 0$, is continuous, bounded and concave, and has a bounded least maximizer defined by $\lambda^* = \min\{\lambda : r(\lambda) = \max_{\lambda \geq 0} r(\lambda)\}$.

when $F_t(.)$ changes over time. Zhao and Zheng show that (P.II) holds in their model under the following sufficient condition on the reservation price distribution: For any p_1, p_2 with $p_1 > p_2$, the conditional probability that the customer is willing to pay p_1 given that she would buy at p_2, i.e., $(1 - F_t(p_1))/(1 - F_t(p_2))$ is increasing in t. In other words, (P.II) holds if the probability that a customer is willing to pay a premium decreases over time.

One might imagine that the price/demand dynamic between two successive purchases in [28] is similar to the one in the multi-period model of [46] where there is a single good, T goes to infinity and each time period is infinitesimally small. In both cases, the price decreases (until a sale occurs), as the remaining time horizon for selling the good becomes shorter. However, notice that for each time period where the good is unsold, the seller can update her belief about customers' valuations in [46] (due to the assumption of equal customer valuations), whereas there is no demand learning in [28] due to the assumption of Poisson arrivals. In [28], the price decrease is a result of the decreasing expected profit one can make in the remaining shorter time horizon.

The optimal pricing policies in [28] and [8] call for the continuous updating of prices over time, which is not practical. Therefore, Gallego and Van Ryzin look at the following *fixed price* heuristics with more "stable" prices: (*i*) FP: Set the price to $\max\{p^0, p^*\}$, and (*ii*) OFP: Set the price to the "optimum" fixed price that maximizes expected revenue. To test the effectiveness of their heuristics, they first find an upper bound to the optimum solution using a deterministic demand function. They show that both FP and OFP are asymptotically optimal if: (1) the number of items for sale is very large and there is plenty of time to sell them, (2) there is the potential for a large number of sales at revenue maximizing prices ($\lambda^* T >> 1$) and there are enough items in stock to satisfy this potential demand ($n \geq \lambda^* T$).

Although theoretically interesting, the conditions for the asymptotic optimality of the FP and OFP heuristics are very unlikely to be satisfied in practice. Therefore, in addition to continuous price paths and simple heuristics, Gallego and Van Ryzin [28] consider the case where prices have to be chosen from a discrete set of allowable prices $p_1 < p_2 < \ldots < p_k$. The deterministic solution (which is asymptotically optimal) in this case has two price points: some p_{k^*} for a specified period of time, and a neighboring price p_{k^*+1} for the rest of the horizon. This result parallels that shown in [21].

Motivated from the practical applicability of few price changes, Feng and Gallego [24] and Bitran and Mondschein [8] study models to allow for only a fixed number of price changes during the season. In Feng and Gallego, the number of price changes is limited to one, the set of allowable prices, as well as the initial price, are inputs to the problem and the goal is to find the optimal timing of the price change. They study the markdown and markup problems, where the second price has to be lower and higher, respectively, than the initial price. They also study the general price policy where one has to decide whether the price should increase (e.g., as in airlines) or decrease (e.g., as in retail). They show that the optimal policy is to decrease (increase) the initial price as soon as the time left falls below (above) a time threshold, which is an increasing function of the remaining stock size. These results are extended to more than two prices in Feng and Xiao [25], who derive the optimal

solution in analytical form.

Bitran and Mondschein [8] look at periodic pricing policies where price can be modified at most K times during the planning horizon and the length of each period (time between two successive price changes) is given. They model this problem as a dynamic program, which requires the solution of a nonlinear program at each stage (they use the Fibonacci algorithm to solve the NLP). They show that the constant pricing policy is optimal (in the limit) when the capacity goes to infinity and the reservation price distribution is invariant with time. They present results from computational experiments with monotonically decreasing price paths, where the planning horizon is two weeks, prices are updated four times, and the arrival rate is 70 customers. They observe that there is only a small gap (less than 2.2%) between continuous and periodic pricing policies. They also observe that a higher variance in reservation prices leads to a higher initial price and a higher percentage of price reduction. These observations are in line with the observations of Lazear about the impact of reservation price variability on prices. When the variance of the reservation prices is high and the initial inventories are "small," they observe higher profits (compared to the low variance case). But if the initial inventory is high, the situation is reversed: as the inventory goes to infinity, a larger variance in reservation prices yields a smaller total expected profit and a higher number of unsold units.

In addition to truly dynamic pricing policies, Bitran and Modschein also consider pricing policies with announced discounts. The policy consists of an initial price and a fixed discount per period (e.g., the practice of Filene's Basement, a large department store in Boston). Given the discount factor (for each period), the only decision to be made is the initial price. They do computational testing with four price updates and a time-invariant reservation price distribution, observing that (i) the loss in expected profits (compared to the optimal policy with at most K price changes) when implementing this type of policy can be significant, and (ii) for all fixed discount policies, the initial price is lower (higher) than the initial price of the optimal pricing policy below (above) a certain inventory level. They also show (ii) theoretically in the limit as the inventory goes to infinity. A similar policy with announced discounts is studied in [21]; however their policy is more general in that the seller can choose all the prices, not only the initial price.

The single-store models discussed above are generalized by Bitran et al. [7] where prices and inventories are coordinated among multiple stores of a retail chain. Common practices for allocating inventories to stores include: (1) an initial allocation without further redistributions, (2) an initial assignment and further reallocations to respond to demand/sales imbalances, (3) distribution from a central warehouse on a periodic basis according to sales patterns. The authors focus on the first two models and on pricing policies for clearance markdown sales. Both the no-inventory-transfer and the inventory transfer situations are modeled as dynamic programs. Since the large state space of these formulations does not allow the solution of most practical problems, the authors develop heuristics to find approximate solutions. The heuristics are based on (near) myopic solution approaches, trying to find the single best price for the rest of the horizon in each period. The heuristics are tested on randomly generated data with two stores and five periods in which prices are revised,

as well as on real data from a Chilean retail chain with eight stores. The results show that the heuristics' performances are close to the optimum, and significantly better than what is currently used in practice. The authors also develop a simulation game, which allows managers to test different pricing policies on randomly generated demand scenarios.

The papers discussed so far model demand as a function of price (and time) only and assume that the initial inventory level is exogenously determined. Smith and Achabal [67] and Mantrala and Rao [49] consider initial inventory decisions in conjunction with markdown pricing decisions.

Smith and Achabal model the demand for a seasonal good as a function of the current on-hand inventory level, as well as time and price. The goal is to determine the optimal initial inventory level I_0 and the optimal price path for a product over its relatively short lifespan to maximize the retailer's profits. Given their focus on seasonal or fashion merchandise, the authors consider only clearance (markdown) pricing policies.

By solving for the optimal price path in closed form, the authors find that: (i) the optimal price at any time t should exactly compensate any reduction in sales due to inventory levels below the critical minimum level f_0; (ii) there are six possible optimal terminal price and inventory commitment (p_e, I_0) pairs. In all cases, except one, it is optimal for the retailer to set the terminal price to clear all of her inventory; (iii) as the sales rate becomes more sensitive to the inventory on-hand, the optimal initial price increases but the optimal price path declines more steeply; and (iv) a larger initial inventory level postpones the time at which sales are adversely affected by low inventory levels, and hence postpones price markdowns until later in the season.

Although the optimal price path is continuous under this model, in implementing their policies at three major retail chains, the authors use a discretized version of the optimal price path, employing at most two markdowns in price. The experience of the three retailers was mixed, where the degree of success was closely aligned with the accuracy of the retailer's inventory data. The modeling and solution approaches proposed by Smith and Achabal, in particular, the demand model, are currently being used in the decision-support tools offered by SpotLight Solutions.

Mantrala and Rao [49] discuss a decision-support system called MARK, that aids retailers in initial inventory and markdown pricing decisions. The software has two modules, which consider the costs of holding inventory, stockouts, customer returns and price changes. Given a discrete set of permissible prices for each period t and a fixed initial inventory, Module 1 solves for optimal unconstrained pricing (OU), where the prices are free to go up and down, or optimal markdown-only pricing (OM) problems by modeling them as finite-horizon stochastic dynamic programs. Module 2 determines the optimal initial inventory (OI) in conjunction with optimal OU or OM pricing, by iteratively solving OU or OM at each point of a discretized range of inventory. Given a predetermined plan for the timing and magnitude of markdowns (GP), Module 2 can also determine the optimal inventory taking into account the vendor's constant unit price or quantity-discount schedule and any fixed merchandise procurement budget constraint. Demand in period t at price P_{tj} is modeled by

$D_{tj} = \alpha_t M (P_f/P_{tj})^{\gamma(t)} \epsilon_t$, where P_f is the full price at the beginning of the selling season, M is the total season demand at price P_f, α_t is the proportion of season's demand that materializes in period t ($\sum \alpha_t = 1$), $\gamma(t)$ is the demand elasticity in period t, and ϵ_t is a log-normally distributed random variable to model random disturbance in period t.

Using data from a local store of a large US retail chain, Mantrala and Rao report computational results on the performance of MARK on determining the initial inventory and markdown schedule for a new style of men's walking shorts. Computational results show that (i) a constant price policy is likely to be more profitable than an automatic markdown policy (e.g., half price sale after half season), which does not take into account the anticipated pattern of demand, and (ii) OM policy yields the highest profits following the OU policy. Next, the authors address the question of choosing the initial inventory level and how it interacts with the choice and the performance of the dynamic pricing policy. They find that the expected profits of optimal markdown policies are significantly higher than the profits of the automatic and fixed price policies with optimized inventory. Furthermore, these profits are achieved with lower levels of inventory. The profits under nonoptimal policies are quite sensitive to the choice of the initial inventory whereas the profit curves of OU and OM are rather flat in the region around the optimal inventory. This result suggests that the optimal pricing policies help compensate the profit loss due to the errors in setting the initial inventory. In summary, the amount of initial inventory depends on the pricing policy and a company can obtain highest profits by jointly deciding on the initial inventory and pricing policies. MARK is available commercially under the name B_Line from Mantrala Associates (see Section 3.2).

The papers discussed so far in this section assume myopic customer behavior. In contrast, Emaghraby, Gülcü and Keskinocak [21] consider strategic customer behavior in designing pricing policies. The authors analyze the optimal design of a markdown pricing mechanism, where the seller who wants to sell a fixed inventory posts a decreasing price schedule and at each price step, buyers decide how much, if any, to purchase. Acting strategically, a buyer might choose not to purchase at a given price step even if that price is below his valuation, hoping to purchase at a later (lower) price step to increase his surplus. Under the assumptions of their model, the authors show that using two price steps in the markdown mechanism is optimal.

The strategic behavior of customers in Elmaghraby et al. is motivated by the fact that sales take place over a very short selling horizon (at most a few days), all potential customers are available at the beginning of the selling horizon, and customers can observe future as well as current prices. Lazear briefly touches upon the issue of strategic buyers, and concludes that under his modeling assumptions, strategic behavior would not really benefit a buyer if the number of buyers, and hence, the competition, is high. However, he points out that strategic behavior would be important if a small number of buyers compete over a limited supply of goods, which is the case in [21].

Most of the research results discussed in this section indicate that in general a firm can achieve significant potential benefits from dynamic pricing even with a limited number of price changes. Given the high cost of changing prices in brick-and-mortar stores or catalog

sales, these results are very encouraging. They suggest the applicability and potential benefits of dynamic pricing in traditional channels, as well as in online channels.

3.2   Software Tools for Dynamically Pricing a Fixed Inventory

For most short life cycle items, we see two types of markdowns in practice: (1) temporary markdowns (or sales) in which a fixed discount is offered over a fixed period of time and then prices return to their initial level, and (2) permanent (or clearance) markdowns in which the next price can only be lower than the current one. The majority of software products designed to address pricing decisions for short selling seasons focus on clearance (or markdown) pricing. Companies that currently sell markdown software include i2, SpotLight Solutions, ProfitLogic and Mantrala Associates. The key business problems addressed by these tools are which products to clear, and what discounts to offer and when. The pricing software B_line offered by Mantrala Associates also has the capability to jointly optimize pricing and initial inventory decisions (B_line is discussed under the name MARK in [49]; see Section 3.1).

Markdown price optimization tools take as input a model of demand (as a function of price), business rules specified by the retailers, current inventory, desired ending inventory, a sell-thru date and various costs (e.g., inventory-carrying costs, cost-of-capital, cost of implementing price-changes, and salvage-values).

Business rules are constraints such as: (1) The allowed number and frequency of markdowns. For example, there should be at least a week between two consecutive markdowns. (2) Min-max discount levels or maximum lifetime discount. (3) The minimum number of weeks before an initial markdown can occur. (4) The types of markdowns allowed (e.g., 10%, 25%, etc.). Equivalently, the set of permissible prices. (5) The "family" of items that must be marked down together (e.g., sets of tops and bottoms or items on the same rack).

For example, i2's markdown pricing tool allows the user to specify the number and magnitude of the discounts and then optimizes their timing. Similarly, Mantrala Associates' software takes as input a set of permissible prices and finds the optimal unconstrained or markdown pricing policy. Motivated from practical applications, some of the academic papers also consider similar types of business rules that impose constraints on pricing policies. For example, Gallego and Van Ryzin [28] consider the case where prices have to be chosen from a discrete set of allowable prices. Bitran and Mondschein [8] and Feng and Gallego [24] allow only a fixed number of price changes during the selling horizon, which may be important if the price changes are costly or if frequent price changes could negatively impact the image of the company and its long-term profitability.

Given the reluctance of most retailers to completely entrust pricing decisions to a decision support tool, these tools are commonly used for "what-if" analysis, allowing the retailers to test and analyze the profitability of various scenarios and to explore new pricing scenarios. For example, Spotlight Solutions' software allows its customers to see what the 'single best' price is, and then solve for the optimal prices given that there be, for example, at least two

markdowns. By solving for the optimal single price, retailers are able to see the potential benefits from adopting more flexible pricing strategies. Retailers also receive feedback on the benefits of introducing new price points, and identify redundant ones (e.g., a 25% and 30% percent markdown result in the same customer response).

3.3   Bridging the Gap

It is encouraging to see that the academic research on dynamic pricing of products without inventory replenishment has made some impact in practice, e.g., the models in [67] have been incorporated into the software of SpotLight Solutions. However, such impact has been limited and there is a gap between the type of models needed in practice and what is available in the literature. Furthermore, advances in e-commerce and information technology (IT) offer new opportunities for dynamic pricing and pose new and interesting research questions. We conclude this section by discussing four possible directions for future research: (1) the consideration of multiple products and multiple stores/sales channels; (2) endogenizing the salvage value; (3) endogenizing initial inventory; (4) strategic customers; and (5) competitors' pricing decisions. Some of these research directions are also important for pricing decisions with inventory replenishment, a topic discussed in the following section.

Multiple Products All the papers we reviewed that study dynamic pricing in an NR-I market setting consider only a single product. Several papers in the literature consider pricing multiple products simultaneously, however their focus is mainly on static product-line pricing, not dynamic pricing [64]. Product dependencies in pricing are important not only for setting regular prices, but also for deciding on markdowns. For example, given a set of matching tops and bottoms, should the seller markdown both, or only markdown one, hoping that the increased demand for the marked down item will also increase the demand for the matching item? One reason why retailers mark down items is to open up shelf space for new arrivals. With the availability of the Internet channel, one possibility for a retailer is to move its excess inventory to a central depot and sell online. This would greatly reduce the need to markdown due to limited shelf space, and hence, might help to increase the length of the selling season for an item. The need for considering pricing decisions of multiple products simultaneously when these products are complements or substitutes is further discussed in Section 4.3.

Multiple Stores One of the most important missing links between the academic literature and the 'real world' is the need for dynamic pricing strategies that consider multiple sales channels, with possibly different demand patterns, simultaneously. Since different sales channels, such as stores in different locations or Internet versus bricks-and-mortar stores, can experience different demand patterns for the same product, pricing decisions need to incorporate multiple products and sales channels and generate a price schedule per item per store. The existence of multiple stores also offers the seller the opportunity to move inventory around between stores and enriches the optimization problem facing the seller. If a product is selling well in one store but not in another, should the seller drop the price on the slow-selling store or transfer inventory? Rather than steeply marking down excess

inventory in bricks-and-mortar stores, should the seller move all of its excess inventory to a central depot that replenishes the online store and sell those items online? Having the excess inventory in one location might provide a more complete assortment (with different colors and sizes), rather than having a few items dispersed over many stores, which might positively affect the demand. On the other hand, relocating inventory is costly, and the seller needs to tradeoff the benefit of moving inventory against the cost.

Salvage Value Most of the papers reviewed in this section, as well as the software tools, take as input a salvage value for the items that remain unsold at the end of the selling season. Current pricing models could be extended such that the salvage value is also a decision variable, depending on the choice of the seller among multiple liquidation channels. According to Mantrala and Rao [49] "... any remaining inventory ... would be removed to make room for the next season's merchandise. Although the store could later donate these lefover units to charity and receive a tax deduction, the store managers treated unsold goods at season's end as having zero salvage value ..." While the outlets for leftover units have been fairly limited in the past, the Internet has opened up many opportunities for inventory liquidation. One possibility, as we discussed above, is to sell such inventory through the retailer's own website. Alternatively, a number of online sites such as www.overstock.com specialize in selling excess inventory from multiple manufacturers. The retailer's choice for disposing of excess inventory will impact the salvage value, which, in turn, will impact the optimal dynamic pricing policy.

Initial Inventory While optimizing markdowns protects margins and clears the sales floor, buying closer to what the company will sell at its planned margin and allocating merchandise to stores based on forecasted demand are equally important [31]. Initial inventory decisions are considered in some papers (e.g., [49] [67]), and simultaneous optimization of pricing and initial inventory decisions is an area that future research should address. For example, if one can choose the inventory optimally for a given pricing policy, does the benefit of having an optimal pricing policy over a heuristic policy decrease or increase? How sensitive are the profits of optimal and heuristic inventory policies to deviations of the initial inventory from the optimal level [49]? Software solutions for initial inventory planning in conjunction with pricing decisions include B_line by Mantrala Associates and Buying4Profit by ProfitLogic. ProfitLogic also offers a product, Allocating4Profit, for allocation of inventory to multiple locations.

Strategic Customers An important element that is largely missing both in most of the academic literature and price optimization software is the consideration of strategic customer behavior. While there are many papers (not relevant for this survey) that examine dynamic pricing in the presence of strategic customers [4], these papers are devoted to stylistic models where inventory considerations are ignored. One could argue that for short life cycle products, a customer can observe the pricing policies of a seller over time and try to time his purchases to maximize his expected utility. Following price changes is especially easy for customers who are shopping from online stores. In the future, one can envision that customers will have bots (software agents) that will track prices of products for them. The data collected by these bots can then be plotted to observe the shape of the price paths of

the products. For example, if a customer observes that a merchant regularly marks down items after they are sold at the initial price for two weeks, he might decide to act strategically and wait for the first markdown to make his purchases. One can further imagine that intelligent decision-support tools for individual customers can analyze price data collected by the bots and make suggestions to the customers about the timing of their purchases[4]. An interesting but equally challenging research direction would be to incorporate into the models customers' strategic purchasing behavior in response to the firm's pricing strategy.

Competitors' Pricing Decisions In a competitive business environment, consumers' purchasing decisions take into account prices offered by competing firms. Information technology allows companies to track competitors' prices automatically and incorporate that information into their pricing decisions. For example, Buy.com Inc. developed technology using software "bots" to monitor prices on competing sites such as Amazon.com and Best Buy [35]. Competitors' prices, along with other information, are then fed into the dynamic-pricing software from KhiMetrics, which suggests price changes on Buy.com. Hence, competitors' pricing decisions need to be considered while developing a dynamic pricing policy, an important element missing from the current literature.

4 Market Type R-I-M: Inventory Replenishment, Independent Demand and Myopic Customers

In this section, we shift our focus from short-term pricing decisions of products with fixed inventory to the day-to-day operations interplay between pricing and procurement when inventory can be replenished. For a large portion of the nondurable products sold in retail markets, such as consumer package goods (CPGs) and fresh produce, the pressing question facing managers is how to coordinate pricing with inventory procurement/production decisions. The impact of these decisions on a sellers profits is inextricably linked. For example, setting the price of a product too low could lead to stockouts and lost sales at a potentially higher price while waiting for inventory replenishment. Conversely, setting the price too high could lead to slow-moving or excess inventory and high holding costs. We provide a brief overview of the literature on dynamic pricing with replenishment (a more detailed discussion is available in the online Appendix), discuss the nature of the DPOSPs' products and conclude with future research directions.

4.1 Literature Review

The rich literature on inventory management [39] [77] [85] typically assumes that the price for a product is a static single price and is exogenous to the inventory management problem.

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[4]Some researchers have begun to examine the role of bots in e-commerce [40]. However their studies have ignored inventory considerations and have assumed that the software agents behave myopically, i.e., they compare prices across online retailers at any one point in time, but do not consider the future price path when making a purchase decision.

In contrast, the papers reviewed here allow price to also be a decision variable and vary from period to period. In all of these papers, the seller is a monopolist, selling a single product in a multi-period setting, and faces a demand that is not dependent on sales in previous periods. The seller's problem is to determine an inventory and pricing policy with the goal of balancing demand and inventory and maximizing profits.

The literature in this category can be divided into three groups based on modeling assumptions. In first group of papers ([22] [75] and [83]), the seller faces an *uncertain* demand, has *convex* production, holding and ordering costs and *unlimited* production capacity. The second group extends this model by incorporating (*i*) a fixed ordering cost ([74] [14]) and (*ii*) *limited* production capacity [13]. The third group focuses on models where the seller faces a *deterministic* demand ([5] [59]).

The first group of papers ([22] [75] and [83]) address the optimal inventory and pricing policy of a seller who faces an uncertain demand where prices are changed periodically (as opposed to continuously) over time. In each period, before demand is realized, the seller must decide how much inventory, y_t, to have on hand at the start of the period. The seller faces an uncertain demand in each period that is only a function of price $D(p_t)$, and incurs three possible types of costs in period t: a convex production cost, a convex holding cost and a convex ordering cost. [5]

By considering variations on demand, cost structure, lost sales or backlogging, and production leadtime, all three papers find that a base stock list price (BSLP) policy is optimal for a wide range of settings. A BSLP policy is defined as follows: if the inventory at the start of period t, x_t, is less than some base stock level y_t^*, produce enough to bring the inventory level up to y_t^* and charge p_t^*, (*ii*) if $x_t > y_t^*$, produce nothing and offer the product at a discounted price of $p_t^*(x_t)$, where $p_t^*(x_t)$ is decreasing in x_t.

Building on the earlier work of Karlin and Carr [39][6], Zabel [83] considers a setting where the seller faces a finite selling horizon ($T < \infty$), production orders are filled instantaneously, the production cost $c(q_t)$ is convex in q_t and holding costs are linear, and unmet demand is lost. When the seller faces a downward sloping linear demand curve with an additive noise term η_t, Zabel finds that (*i*) the optimal price p_t^* is a decreasing function of the on-hand inventory y_t; (*ii*) given an on-hand inventory level of y, the optimal price with t periods left is greater than with $t-1$ periods left, i.e., $p_t^*(y) > p_{t-1}^*(y)$; (*iii*) the optimal amount to produce is decreasing in t for any given x, i.e., $y_t^*(x) > y_{t-1}^*(x)$; and (*iv*) the critical level x_t^* is decreasing in t. It is interesting to note that results (*i*) and (*ii*) are identical to the ones derived by Gallego and Van Ryzin [28] in the non-replenishment settings.

Thowsen [75] extends Zabel's analysis to the case where the seller can allow for backorders, incorporates the possible deterioration of inventory over time and allows for the possibility that payments for demand are not received until after the order is placed. Thowsen finds

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[5]The demand models used can be view as "lumping" together the individual (Poisson) arrivals in [28] and [24] over the length of a period, such that all of the demand that occurs in the same period sees the same price.

[6]Karlin and Carr limit their analysis to either a one-period case or an infinite horizon case where only a single static price must be chosen.

that a BSLP policy is optimal when all of demand is backlogged, production costs are linear, holding and stockout costs are convex, and $E[\eta] < \infty$. Thowsen also finds a BSLP policy to be optimal in the event that only partial backlogging is allowed, provided that production and stockout costs are linear, holding costs are convex and η is drawn from a family of distributions where $E[\eta] = 0$.

Federgruen and Heching [22] build on Thowsen's results by considering cases where (a) prices are only allowed to decrease over time and (b) $T = \infty$. When prices are only allowed to decrease over time, they find that the optimal inventory and pricing policy is a *modified* BSLP, provided that holding and backlogging costs are convex and variable production costs are constant. That is, the optimal policy is of the form: (i) if the price in period $t+1$, p_{t+1}, is greater than or equal to p_t^*, implement the BSLP policy (as described above), (ii) if $p_{t+1} < p_t^*$, find the optimal inventory level $\hat{y}(p_{t+1})$ corresponding to a price of p_{t+1}. If the on-hand inventory at the start of period t (x_t) is less than or equal to $\hat{y}(p_{t+1})$, bring the inventory level up to $\hat{y}(p_{t+1})$ and charge price p_{t+1}. If $x_t > \hat{y}(p_{t+1})$, then produce nothing and charge price $p^*(x) \leq p_{t+1}$. When $T = \infty$ and the seller wishes to maximize expected discounted profits, [22] find that a BSLP is optimal. If the seller wishes to maximize average long run profits, then the optimal pricing policy depends on whether prices are allowed to move freely or are restricted to decrease over time. They find that when prices are allowed to move freely, then a BSLP policy is optimal. However, if the seller can only decrease prices over time, then her optimal strategy is to charge a fixed (static) price p' in all periods and follow a simple order-up-to policy with order-up-to-level $y^*(p')$.

While the papers above assumed that all costs are convex, Thomas [74] and Chen and Simchi-Levi [14] allow there to be a fixed component to ordering costs. Thomas conjectures, and Chen and Simchi-Levi prove, that an (s, S, p) policy is optimal when (i) demand is additive in a finite time horizon and (ii) the seller's objective is to either maximize expected discounted profits or maximize average long run profits in the infinite time horizon model. Under an (s, S, p) policy, whenever the on-hand inventory at the start of period t, x_t goes below s, the seller replenishes up-to level S; if $x_t > s$, the seller orders nothing and charges price $p(x_t)$. An (s, S, p) policy is similar to a BSLP policy, with the exception that the presence of fixed ordering costs implies that there is a maximum inventory level above which it is not profitable to reorder, but instead, better to manage demand uncertainty using flexible pricing and existing inventory. Interesting they find that the optimal price schedule p is not necessarily a non-increasing function of x_t. For more general demand functions, such as multiplicative demand, *k-concavity* conditions are violated, and a (s, S, p) policy may no longer be optimal. Employing a more general concept of concavity, i.e., symmetric k-concavity, they show that a (s, S, A, p) policy is optimal. Under a (s, S, A, p) policy, (i) if the inventory at the start of period t, x_t is less than s_t or if $x_t \in A_t$, where $A_t \subset [s_t, \frac{s_t + S_t}{2}]$ then an order of size $S_t - x_t$ is made and the seller sets price equal to $p(S_t)$. Otherwise, no order is placed and the seller set a price of $p(x_t)$.

The five papers above assumed that the seller did not have any external constraints on the number of units it could produce in any time period. However, the presence of capacity constraints is a real (and often binding) constraint in many production or retail settings.

When capacity is bounded, a seller can use price as a means to hoard inventory during 'weak' demand periods in order to have sufficient inventory during 'strong' demand periods, and may choose to produce for inventory and turn away demand in a given period. Chan et al. [13] analyze the structure and performance of *partial planning* strategies in the presence of stochastic demand and capacity constraints. Under a partial planning strategy, the seller selects a price or production schedule for the entire (finite) planning horizon at $t = 1$, and the remaining decision (price or production) is used to manage demand uncertainty and inventory costs. Using numerical examples, the authors demonstrate that when the seller is able to employ discretionary sales (i.e., withhold available inventory from customers), the benefits of dynamic pricing (as opposed to fixed pricing) tend to increase as (i) capacity becomes more constrained and (ii) demand seasonality increases. As with Chen and Simchi-Levi, Chan et al. find that the optimal price in period t, p_t is not necessarily decreasing in x_t. This result stems from the non-concavity of the expected profit to go function.

In the above papers, the seller used price as an instrument to manage the uncertainty associated with demand. In contrast, Rajan et al. [59] and Biller et al. [5] study the use of dynamic pricing in the presence of *deterministic* demand. Rajan et al.[59] focus on price changes that occur within an order cycle when the seller sells a single *perishable* good such as fresh produce. The seller orders a new shipment of inventory every T periods, which is delivered instantaneously. The deterministic demand for the product is a decreasing function of the age of the product (i.e., the time elapsed since the beginning of the order cycle) as well as price. Given the deterministic demand and instantaneous replenishment rate, the seller will deplete her entire inventory within each order cycle (i.e., the seller incurs no lost sales or backorders). Rajan et al. assume that there are four types of costs associated with inventory; a fixed ordering cost; a constant variable ordering cost; a constant holding cost per unit per time period; and a wastage cost associated with inventory decay over time. The problem facing the seller is to determine the optimal price path within an order cycle, the optimal cycle length T and the optimal order quantity Q, so as to maximize her average profits over time (assuming no discounting). Rajan et al. find that, (i) the optimal price to charge t periods after the last order was placed, i.e., p_t^*, is independent of T; and (ii) the optimal price path p_t^* is unique. Furthermore, the optimal price may be increasing, decreasing or both in t. The fact that prices may increase or decrease over an order cycle stems from the behavior of the costs over time and the rate at which demand diminishes as the product's age increases. As t increases, so does the total unit cost due to wastage cost component, which would tend to make prices rise over time. However, this upward pressure is countered by a downward pressure arising from decreasing demand over time. If inventory decays at a high rate, then the upward pressure may dominate and increasing prices prevail. If, on the other hand, demand decreases sharply over time, the seller may find it optimal to decrease prices over time so as to attract customers later in the cycle.

Rajan et al. go on to characterize the profit difference between a dynamic and the optimal fixed pricing policy. While unable to characterize this difference under general settings, they are able to do so when prices are either monotonically increasing or decreasing over the cycle. When p_t^* is decreasing in t, they find that a dynamic pricing policy performs significantly better than the optimal fixed pricing policy when demand is high. Likewise,

when p_t^* is increasing in t, a dynamic pricing policy performs significantly better than the optimal fixed pricing policy when inventory costs are high.

Biller et al. [5] also consider the pricing problem for a single manufacturer (seller) who faces a deterministic demand (i.e., there is a one-to-one relationship between sales/demand and price in period t). Although the seller has perfect foresight of demand over the relevant time horizon, she has a capacity constraint on the number of units she can produce in each period. While the seller is unable to backorder demand, she is able to stock inventory for future demand. The seller's problem is to determine the optimal (discrete) price path (in essence determining the number of units sold in each period) and production quantities over $T < \infty$ time periods in the face of production capacity constraints. The authors characterize the following "greedy" algorithm that solves the seller's problem to optimality provided that the revenue functions in each period are concave with respect to sales: Select a period in which to increase sales by one unit, so that the total profit contribution is maximized (the best production period for that unit of sales can be determined via a network algorithm); then select the next best period in which to increase sales by one unit, and so forth. The authors show that the seller's problem can be solved to optimality by a greedy algorithm provided that the revenue functions in each period are concave with respect to sales (this would be the case, for example, when the demand is a linear function of price). Using numerical experiments, the authors test the performance of the optimal dynamic pricing policy against the optimal single-price policy, under a variety of demand scenarios. Assuming a binding capacity constraint (i.e., at certain time periods, the optimal sales level is bounded above by the presence of capacity constraints), they consider patterns of seasonal demand, and analyze the resulting variability in sales, price, profit, and frequency of price changes. From their experiments, the authors conclude that (i) dynamic pricing reduces sales variability significantly (over single pricing), (ii) the benefits of dynamic pricing are the greatest when demand is initially high and decreases over time, and conversely (iii) the benefits of dynamic pricing are the smallest when demand is initially low and increases over time.

4.2   Category Pricing

For many retailers today, the technology is becoming available to allow them to change products' prices frequently. The question then becomes, how should a product's price change dynamically over time and how much should a retailer stock of a product? For non-durable goods, a typical retailer must make pricing and inventory decisions for hundreds or thousands of products (e.g., there can be as many as 30,000-50,000 SKUs in a grocery or drug store with items possibly priced differently across various stores). The substitutability between products and the resulting interdependence of their demands compounds the difficulty of the pricing/inventory decision. One can argue that all products' prices are somewhat interdependent and pricing decisions should consider all the products offered by a firm as well as its competitors simultaneously. However, the data requirements necessary to sustain such a comprehensive pricing scheme are enormous, while the relative benefits of including the marginally dependent products are small. One reasonable approach to assuage the informational burden is to identify *families* or categories of products whose demands

are significantly dependent on each other and simultaneously consider pricing decisions for the products in the same family.

In the last few years, a few DPOSP who offer *category pricing* tools have entered the U.S. market, notably, DemandTec and Khimetrics[7]. Category pricing tools are used to determine "regular" prices (as opposed to clearance prices) for families of products that are close substitutes (a cluster or category of items) and that are typically managed together.

Inputs to a category pricing tool include SKU level price-demand sensitivities, item-links (i.e., families of products), competitor-prices, price image (competitive positioning) constraints, planned promotional events, seasonality effects and threshold psychology effects (e.g., the effect on demand of seeing a price tag of $12.99 versus $13.00). The first task facing DPOSPs is to identify groups of products for which product line-size parity must be maintained. For example, in setting the price for a 10 oz. bottle of ketchup, the retailer must also take into account the prices of the 16 oz. and 24 oz. bottles. Once product families have been identified, the next step is to find an appropriate model for demand. The DPOSPs use demand estimation models based on an "attribute management" system, where items with similar attributes are collected in the same cluster. There typically is a "library" of different types of price-demand functions and a best fit is found, using Bayesian econometric modeling, for an item depending on into which cluster it falls. This is done by testing the model on historical data (from the past 1-2 years) when available and drawing from similar products otherwise. When the data is available, demand estimation also takes into account the existence of competitors and their prices, seasonality, macroeconomic factors, and where an item is in its life cycle. The resulting demand model is then validated and its parameters updated frequently throughout the selling horizon as actual sales unfold.

Item link constraints are used to maintain price consistency across comparable items. Price image constraints take into account required price consistency with respect to competitors' prices. For example, a price consistency constraint could consist of the maximum price difference permissible between the top national brand and the store brand (price band). Category pricing tools also take into account business rules and policies, which include allowed number and frequency of price changes or the minimum number of weeks before a price change can occur.

A retailer's objective in setting a products price might vary depending on the type of product. For some items, such as a specialty coffee, the objective is to maximize profit margins. Conversely, for other items, such as milk, the objective is to increase the traffic through a store. Both DPOSPs offer tools that allow the user specify different objective functions when determining the optimal pricing policy. Given the desired objective, the tool outputs static prices for price families, i.e., a price envelope comprised of an array of prices for products within a family, as well as sales forecasts.

A key differentiator between DemandTec's and Khimetrics software is that DemandTec's

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[7]Knowledge Support Systems offers comparable pricing tools in the UK. Supply chain management companies i2, Manugistics and Retek have also recently entered the U.S. market but their category pricing tools are still in the development stages.

tool uses an activity based cost (ABC) model to accurately capture the cost of having an individual product on the retailer's shelf. An ABC model tries to estimate the cost of inventory throughout the supply chain by tracking each time a cost-incurring action takes place for a product (placing an order, receiving an order, stocking the product, shipping the product, loading/unloading from distribution center, etc.). For example, depending on whether the product is received in individual cases or in pallets, the costs of loading and unloading will be different due to the number of labor hours that are required to process the order. Whether or not the product has special needs for storage (e.g., storing ice cream requires an expensive cooler, while storing toilet paper does not) is also incorporated into the ABC model.

To the best of our understanding, both companies model their pricing problem as a nonlinear, deterministic, static optimization problem. Although static in nature, each software is designed to be run frequently throughout the selling season (e.g., daily if pricing over the Internet and weekly if pricing for a brick-and-mortar store) and demand forecasts are updated by actual sales data. Both DPOSPs claim that their software is able to provide pricing at the SKU-store level. However, the actual level of detail of pricing depends greatly on the retailer's information systems and implementation capabilities and the vendor's ability to work with inconsistencies in the retailer's POS data.

4.3 Bridging the Gap

As opposed to the markdown literature and practice, there is almost no overlap between the R-I-M problems addressed by the DPOSPs and the pricing/inventory literature. The biggest distinction is the absence of inventory decision making in category pricing software. Neither DemandTec nor Khimetrics involves or interfaces with a retailer's inventory management systems (most of their clients have in place inventory management systems provided by other companies), nor do their tools take as input any inventory information. This complete decoupling would be understandable 20 years ago, where constraints on the information exchanged and the flexibility in supply chains would have imposed considerable costs on coordinating inventory and pricing policies. Today, however, e-commerce and advances in information technology are tearing down many of these obstacles to coordination.

The DPOSPs argue the benefits from improved inventory management are additive to any benefits from dynamic pricing; therefore, there is no pressing need to incorporate inventory and pricing decisions into one decision support framework. We believe it would be worthwhile to prove or disprove the beliefs of DPOSP concerning the additive benefits of inventory and pricing policies. That is, are the benefits of considering optimal inventory and pricing policies jointly superadditive (are the policies complements in the sense of Shannon and Milgrom [53]), merely additive or are they partial substitutes? If the answer is that the benefits are additive, then it seems plausible that inventory policies and pricing policies can be determined fairly independently. However, if the benefits of considering both policies simultaneously are superadditive or partial substitutes, then it seems clear that DPOSPs must integrate with inventory management systems in order to offer their customers the

greatest profit potential. For example, Zara, an international clothing-retail store, has a twice-a-week delivery schedule that allows it to effectively manage its inventory and avoid discounting prices over time [34]. Therefore, we believe that the relationship between inventory policies (e.g., order replenishment frequency) and price optimization should be explored further.

In addition, there are three extensions to the literature that must be considered to better capture the decisions and opportunities available to sellers today: (*i*) allowing the timing of replenishments to be a decision variable; (*ii*) studying the dynamic pricing of multiple products with replenishment; and (*iii*) incorporating business rules.

When and How to Replenish All the papers reviewed in this section, with the exception of [59], assume an exogenously (fixed) periodic replenishment schedule. Before the advent of EDI systems or the Internet, this assumption was quite reasonable. The limited ability of the retailer to communicate with suppliers in real-time and the suppliers to re-optimize delivery routes and communicate information to other affected parties quickly, forced retailers to establish simple, fixed replenishment dates, that were usually suboptimal policies. Furthermore, a continuous-review inventory policy would have been difficult to implement due to the retailers' lack of accurate inventory information.

Luckily, advances in IT and ecommerce are making the implementation of continuous-review inventory policies possible, as well as increasing the connectivity between members of supply chains [47]. Firms can now potentially decide *when* and *how* to replenish. The questions that these opportunities now pose are, how should a firm coordinate its inventory policy with dynamic pricing? Will the ability to dynamically change prices imply that a periodic inventory review policy suffices, or should retailers exploit the ability to implement continuous-inventory review policies? Will a continuous-review inventory policy lead to more stable prices over time? How should the seller coordinate replenishments across multiple stores [23]?

Multiple Products with Replenishment Previously, due to informational constraints, retailers employed relatively simple pricing rules, such as cost-plus or match-competitor-pricing. These simple rules did not allow retailers to incorporate the dependent nature across multiple products; rather, product dependencies were embodied in business rules (e.g., min-max price differences across comparable items). Advances in information technology provide the retailers with the ability as well as the required data to optimize prices across multiple products, and therefore, we see this as a research direction deserving immediate attention.

A simple model, with even two products whose demands are interdependent, will allow us to explore how to coordinate inventory policies when multiple products share ordering costs. With demand models of the form,

$D_1(p_1, p_2, t) = a - b_1 p_{1t} - b_2 p_{2t} + \epsilon_{1t}$ (Demand for product 1)
$D_2(p_2, p_1, t) = a - \beta_1 p_{1t} - \beta_2 p_{2t} + \epsilon_{2t}$ (Demand for product 2)

we can incorporate situations where products p_1 and p_2 are substitutes ($b_2 < 0$, $\beta_1 < 0$) or complements ($b_2 > 0$, $\beta_1 > 0$). Product 2 may be a complement to product 1 in the traditional sense (e.g., product 2 is diaper wipes and product 1 is diapers), or it may be a

product that is used to increase traffic through a store (e.g., product 2 is milk). With such an extension to Federgruen and Heching [22], we can reexamine (i) whether a base-stock list price policy is still optimal when products share costs; (ii) whether price changes across periods are greater or smaller in magnitude in the presence of complements/substitutes than in their absence; and (iii) how much the seller increases her profitability by considering the interdependent nature of demand. This last point in particular will be relevant when the demand coefficients are small and data requirements pose a burden on the seller. (The reader is directed to [73] for an overview of the literature on multi-product pricing in the absence of inventory considerations.)

By considering an environment with multiple products, researchers can begin to answer how sellers can coordinate inventory and pricing decisions to control for uncertain demand across products. For example, if product 1 has experienced a large positive shock in demand at time t ($\epsilon_{1t} >> 0$) that nearly depletes the seller's inventory of the product, should the seller replenish her inventory of product 1 (only) immediately, or should she reduce her price for product 2 so as to deplete it more quickly and replenish both items simultaneously? This is only one of many situations which can face a seller and in which a coordinated inventory/pricing policy across multiple products increases the seller's flexibility to meet demand in a profitable fashion.

Incorporating Business Rules Another disconnect between most of the academic literature the practice is the incorporation of business rules into pricing decisions. While such rules might appear to add artificial constraints and take the solution away from optimality in the theoretical models, they may be used for strategic purposes in practice. For example, while e-commerce makes it technically possible for sellers to change prices as often as desired, historical pricing behavior and market norms usually imply that there are a limited number of prices that are used in the marketplace. If the seller faces strategic customers, committing to a rule, such as infrequent and small price drops, might encourage purchases earlier in the time horizon and hence increase the seller's profits. Furthermore, commonly accepted business rules might be useful in warding off excessive price competition when there is more than one seller. Hence, it would be interesting to examine the role of practiced business rules on the resulting optimal dynamic pricing policy, customer behavior, and their possible anti-competitive effect in an oligopolistic environment.

5   E-Commerce Opens the Door for Dynamic Pricing Policies

E-commerce has the potential to impact retailers' pricing strategies by enabling sellers to quickly and costlessly change prices over time, and by providing customer data that can be used in developing and implementing informed/sophisticated pricing strategies. Upon reflection, most of the opportunities offered by e-commerce are not limited to online stores; they are applicable to brick-and-mortar (B&M) stores as well. For example, the emergence of Electronic Shelf Labeling Systems allow B&M stores to change prices quickly, the availability of Point of Sales (POS) data provides information about sales, and the implementation of loyalty programs enable B&M stores to monitor purchases of individual

customers over time. Hence, in many ways, the pricing problems and the decisions faced by online and B&M stores are quite similar from a modeling perspective.

There are, however, some characteristics of the online selling environment that truly differentiate it from the B&M environment, posing new and interesting research questions. In this section, we focus on two of these characteristics: the ability to (i) observe customers' browsing and selection process, and (ii) implement customized pricing/promotion/inventory policies, in *real time*.

Gathering Information About Customers' Purchasing Behavior. Up until recently, in B&M retail environments a customer's preferences, past purchasing history, demographic attributes, etc. were largely unknown to a seller. Today, with loyalty programs some stores track individual customers' purchases and use this information for personalized promotions; for example, in some supermarkets customized coupons applicable towards future purchases are printed at the checkout. However, even with new technologies, the information that can be collected about individual customers in B&M stores is still limited compared to online stores. First, a customer's participation in a loyalty program is voluntary[8], whereas many websites require a customer to "sign up" before making a purchase, which requires the customer to enter basic personal information such as name, address, profession, age, etc. This information along with the customer's purchase history is used by the seller to categorize the customer into a particular market segment and to better estimate that segment's demand. Second, in a B&M store the seller cannot systematically observe a particular customer's purchasing decision (what goods he did buy and what goods he looked at but decided not to buy). The online selling environment offers a great advantage in this respect by allowing a seller to observe and analyze a customer's purchasing behavior in real time. Currently, there are a number of companies on the B2C front, e.g. Double Click and I-Behavior, which provide tools for tracking the behavior of customers browsing online catalogs, and for analyzing the collected information (referred to as clickstream data) to better understand not only what is sold, but also what is not sold and why.

Deciphering the clickstream or price-testing data and translating it into information is not a simple task. The discovery of knowledge from large databases is commonly referred to as *data mining*, and these tools are increasingly used by companies in understanding customers' preferences and in developing pricing strategies. For example, to identify opportunities in steering their most profitable and affluent customers to their own investment offerings, banks such as First Union Corp and Royal Bank of Canada are using sophisticated data mining systems [36]. Some of these data mining problems, e.g., clustering problems, can be modeled as large-scale mathematical programs and have been studied by OR/MS researchers for quite some time [9] [48]. OR/MS researchers could make significant contributions to the theory and practice of pricing by developing improved data mining techniques along with other mathematical models for better capturing demand dynamics.

In addition to analyzing and learning from a customer's purchasing behavior, the Internet

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[8]This might not hold true in the future, if one is to believe futuristic science fiction movies such as Minority Report, where customers are identified by their retina upon entering a store, and customized advertisement is then used for marketing!

also provides online merchants with the ability to proactively do price testing to better characterize market demand [1]. Although some companies, such as Amazon.com, have received much negative reaction from their customers about price testing, other companies, including GE, Penske, Hotwire and DHL Worldwide Express, have successfully used it in developing their pricing policies[9]. While offline price tests can cost hundreds of thousands of dollars and take months to complete, the Web enables real-time tests to be done at virtually no cost. For example, a merchant could charge every 50th customer a higher or lower price and track the results over time. However, the decisions as to what prices to use in estimating demand, what products to test, and how frequently to test a new price point are extremely complex, and significantly impact the effectiveness of price testing and the accuracy of the insights obtained about the demand. For example, changing the price too frequently results in very few observations associated with each price point, whereas changing the price too infrequently may require a longer time period over which to test a handful of price points. Given the short-life cycle of many products there is also the question of how long the price-testing vs. implementation horizon should be. Price testing over a longer horizon could lead to a more accurate demand estimation, but leave a shorter remaining selling horizon. Conversely, price testing over a very short horizon could lead to an inaccurate characterization of the demand. As with dynamic pricing, price testing is an arena that could benefit substantially from the optimization and computational expertise of the OR/MS community.

Customization. The ability to collect detailed information about customers' purchasing behavior allows online sellers not only to better understand the demand, but also to customize the purchasing experience for each individual buyer. In particular, the seller can customize the online store (the web pages), which could result in a unique shopping environment with a different assortment of products and prices, for each customer. For example, Dell's Premier Pages for its corporate customers are designed based on the purchasing needs of each buyer and contain customized prices. Similarly, Amazon.com suggests book titles based on customer's past purchases, as well as at which books a customer is currently looking.

Customized pricing (i.e., third degree price discrimination) entails quoting each 'customer-type' a different price, where each customer type possesses a different underlying demand function based on relevant characteristics, such as age, wealth, geographic location, etc. The study of customized pricing dates back to the early 1920's [58] and has long been known to be an effective means of increasing a seller's profit. One form of customized pricing that stands to benefit immensely from the increased informational capabilities of online retail is 'behavioral' price discrimination, which entails charging customers different prices based on their 'behavior' while shopping. This behavior could constitute which websites they visit, in which order they visit them, when they make a purchase and when they do not, what they purchase, etc. Given the immense potential for behavioral price discrimination, it is a burgeoning area of research. There have been several recent papers that analyze one form of behavioral price discrimination, discriminating according to a customer's purchasing history

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[9]As with dynamic pricing, there are now companies, such as Zilliant, that offer price testing software.

[15] [78] [79] [27] [10].

Customizing the inventory (and prices) for each customer could have several benefits. For example, a focused selection of products directly targeted to a customer's needs may decrease search time/cost for the customer and lead to increased sales. Such customization also allows a seller to exercise greater market power by increasing the prices on the customer's most prefered goods. The benefits of this type of pricing/inventory policy must be weighed against the possibility of losing a customer (no purchase occurs) due to imprecise preference information. We are unaware of any literature that addresses this unique aspect of online sales - and believe it to be an important area for future research.

Due to the customization capabilities of online stroes, the products and prices offered to individual customers or customer-types may be selected based on current inventory levels, the customers' characteristics, or a combination of both. To the best of our knowledge, we are unaware of any research that considers the seller's customized pricing decision *dynamically* and in the presence of *inventory* considerations. We believe this to be a promising future direction for research.

References

[1] W.L. Baker, E. Lin, M.V. Marn, and C.C. Zawada. Getting prices right on the web. *The McKinsey Quarterly*, 2, 2001.

[2] F.M. Bass. A new product growth model for consumer durables. *Management Science*, 15:215–226, 1969.

[3] P.P. Belobaba. Airline yield management: An overview of seat inventory control. *Transporation Science*, 21:63–73, 1987.

[4] D. Besanko and W. Winston. Optimal price skimming by a monopolist facing rational consumers. *Management Science*, 36(5):555–567, 1990.

[5] S. Biller, L. Chan, D. Simchi-Levi, and J. Swann. Dynamic pricing and the direct-to-customer model in the automotive industry. To appear in the Electronic Commerce Journal, 2002.

[6] G. Bitran and R. Caldentey. An overview of pricing models for revenue management. *Manufacturing & Service Operations Management*, 2002. to appear.

[7] G.R. Bitran, R. Caldentey, and S.V. Mondschein. Coordinating clearance markdown sales of seasonal products in retail chains. *Operations Research*, 46:609–624, 1998.

[8] G.R. Bitran and S.V. Mondschein. Periodic pricing of seasonal products in retailing. *Management Science*, 43:64–79, 1997.

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[10]Stole [73] provides a nice summary of this work in chapter 6.

[9] P. Bradley, U. Fayyad, and O. Mangasarian. Mathematical programming for data mining: Formulations and challenges. *INFORMS Journal on Computing*, 11:217–238, 1999. http://www.cs.wisc.edu/~olvi/olvi.html.

[10] P. Bremaud. *Point Processes and Queues, Martingale Dynamics*. Springer-Verlag, New York, 1980.

[11] Chakravarty and Martin. Discount pricing policies for inventories subject to declining demand. *Naval Research Logistics*, 36:89–102, 1989.

[12] L. Chan, Z.J. Max Shen, D. Simchi-Levi, and J. Swann. Coordination of pricing and inventory decisions: A survey and classification. working paper, 2003.

[13] L. Chan, D. Simchi-Levi, and J. Swann. Dynamic pricing strategies for manufacturing with stochastic demand and discretionary sales. working paper, 2002.

[14] X. Chen and D. Simchi-Levi. Coordinating inventory control and pricing strategies with random demand and fixed ordering cost. Working Paper, Massachusetts Institute of Technology, 2002.

[15] Y. Chen. Paying customers to switch. *Journal of Economics and Management Strategy*, 6:877–897, 1997.

[16] Y. Chen and D.C. Jain. Dynamic monopoly pricing under a poisson-type uncertain demand. *Journal of Business*, 65:593–614, 1992.

[17] E. Gerstner Conlisk, J. and J. Sobel. Cyclic pricing by a durable goods monopolist. *The Quarterly Journal of Economics*, 99:489–505, 1984.

[18] P. Coy. The power of smart pricing. *Business Week*, 2000.

[19] A. Dhebar and S.S. Oren. Optimal dynamic pricing for expanding networks. *Marketing Science*, 4:336–351, 1985.

[20] R.J. Dolan and A.P. Jeuland. Experience curves and dynamic demand models: Implications for optimal pricing strategies. *Journal of Marketing*, 45:52–62, 1981.

[21] W. Elmaghraby, A. Gulcu, and P. Keskinocak. Optimal markdown mechanisms in the presence of rational customers with multi-unit demands. Georgia Institute of Technology, Working Paper, 2002.

[22] A. Federgruen and A. Heching. Combined pricing and inventory control under uncertainty. *Operations Research*, 47:454–475, 1999.

[23] A. Federgruen and A. Heching. Combined pricing and inventory control under uncertainty. *Manufacturing & Service Operations Management*, 4(4):275–295, 2002.

[24] Y. Feng and G. Gallego. Optimal starting times for end-of-season sales and optimal stopping times for promotional fares. *Management Science*, 41:1371–1391, 1995.

[25] Y. Feng and B. Xiao. Optimal policies of yield management with multiple predetermined prices. *Management Science*, 48:332–343, 2000.

[26] M. Freimer and D. Horsky. Try it you will like it - does consumer learning lead to competitive price promotions. Working Paper, William E. Simon Graduate School of Business, University of Rochester, 2001.

[27] D. Fudenberg and J. Tirole. Customer poaching and brand switching. *RAND Journal of Economics*, 31:634–657, 2001.

[28] G. Gallego and G.J. van Ryzin. Optimal dynamic pricing of inventories with stochastic demand over finite horizon. *Management Science*, 40:999–1020, 1994.

[29] G. Gallego and G.J. van Ryzin. A multiple product dynamic pricing problem with applications to network yield management. *Operations Research*, 45:24–41, 1997.

[30] G. Girard. Retailers bullish on their price optimization strategies and vendors. AMR Research Report, August 20, 2002.

[31] G. Girard. Retailers should start rrm asap. AMR Research Report, June 5, 2002.

[32] E. J. Green and R. H. Porter. Noncooperative collusion under imperfect price information. *Econometrica*, 52:87–100, 1984.

[33] A. Ha. Optimal pricing that coordinates queues with customer-chosen service requirements. *Management Science*, 47:915–930, 2001.

[34] M. Helft. Fashion fast forward   . http://www.business2.com/articles/mag/0,1640,39407,FF.html, 2002.

[35] C.T. Heun. Dynamic pricing boosts bottom line. *Information Week*, Oct 29, 2001. URL: http://www.informationweek.com/story/IWK20011025S0007.

[36] T. Hoffman. Banks turn to it to reclaim most profitable customers. *Computerworld*, 1998.

[37] A.P. Jeuland and R.J. Dolan. An aspect of new product planning: Dynamic pricing. *TIMS Studies in Management Sciences*, 18:1–21, 1982.

[38] S. Kalish. Monopolist pricing with dynamic demand and production cost. *Marketing Science*, 2:135–159, 1983.

[39] S. Karlin and C.R. Carr. Prices and optimal inventory policies. *Studies in Applied Probability and Management Science*, 1962.

[40] J. Kephart, J. Hanson, and A. Greenwald. Dynamic pricing by software agents. Working Paper IBM Institute for Advanced Commerce, 2000.

[41] S.E. Kimes. The basics of yield management. *Cornell H.R.A. Quarterly*, 30:14–19, 1989.

[42] P. Klemperer. Auction theory: A guide to the literature. *Journal of Economic Surveys*, 13:227–286, 1999.

[43] R. Lal and R. Staelin. An approach for developing an optimal discount pricing policy. *Management Science*, 30:1524–1539, 1984.

[44] M. Landsberger and I. Meilijson. Intertemporal price discrimination and sales strategy under incomplete information. *Journal of Economics*, 16:424–430, 1985.

[45] A. Lau and H.S. Lau. The newsboy problem with price-dependent demand distribution. *IIE Transactions*, 20:168–175, 1988.

[46] E.P. Lazear. Retail pricing and clearance sales. *The American Economic Review*, 76:14–32, 1986.

[47] J. Magretta. The power of virtual integration: An interview with dell computer's michael dell. *Harvard Business Review*, 7907, 2001.

[48] O. Mangasarian. Linear and nonlinear separation of patterns by linear programming. *Operations Research*, 13:444–452, 1965.

[49] M.K. Mantrala and S. Rao. A decision-support system that helps retailers decide order quantities and markdowns for fashion goods. *Interfaces*, 31(3):S146–S165, 2001.

[50] R.P. McAfee and J. McMillan. Auctions and bidding. *Journal of Economic Literature*, 25:699–738, 1987.

[51] J.I. McGill and G.J. van Ryzin. Revenue management: Research overview and prospects. *Transportation Science*, 33:233–256, 1999.

[52] A. Merrick. Priced to move: Retailers attempt to get a leg up on markdowns with new software. *The Wall Street Journal*, August 7, 2001.

[53] P. Milgrom and C. Shannon. Monotone comparative statics. *Econometrica*, 62:157–180, 1994.

[54] K.B. Monroe and A.J.D. Bitta. Models for pricing decisions. *Journal of Marketing Research*, 15:413–428, 1978.

[55] T. Nagle. Economic foundations for pricing. *Journal of Business*, 57:s3–s26, 1984.

[56] B.P. Pashigian. Demand uncertainty and sales: A study of fashion and markdown pricing. *The American Economic Review*, 78:936–953, 1988.

[57] B.P. Pashigian and B. Bowen. Why are products sold on sale? explanations of pricing regularities. *The Quarterly Journal of Economics*, 106:1015–1038, 1991.

[58] A. Pigou. *The Economics of Welfare*. Macmillan, 1920. London.

[59] A. Rajan, Rakesh, and R. Steinberg. Dynamic pricing and ordering decisions by a monopolist. *Management Science*, 38:240–262, 1992.

[60] J. Raju. Theoretical models of sales promotions: Contributions, limitations, and a future research agenda. *European Journal of Operations Research*, 85:1–17, 1995.

[61] K. Raman and R. Chatterjee. Optimal monopolist pricing under demand uncertainty in dynamic markets. *Management Science*, 41:144–162, 1995.

[62] R.C. Rao. Pricing research in marketing: The state of the art. *Journal of Business*, 57:S39–S64, 1984.

[63] V. Rao. *Pricing Models in Marketing, in J. Eliashberg, and G.L. Lilien, editors, Marketing.* North-Holland, New York, 1993. V. 5 of Handbooks in Operations Research and Management Science, Chap. 11.

[64] D. Reibstein and H. Gatignon. Optimal product line pricing: The influence of elasticities and cross-elasticities. *Journal of Marketing Research*, 21:259–267, 1984.

[65] B. Robinson and C. Lakhani. Dynamic price models for new product planning. *Management Science*, 21:1113–1122, 1975.

[66] J.J. Rotemberg and G. Saloner. A supergame-theoretic model of price wars during booms. *American Economic Review*, 76:390–407, 1986.

[67] S. Smith and D. Achabal. Clearance pricing and inventory policies for retail chains. *Management Science*, 44:285–300, 1998.

[68] J. Sobel. The timing of sales. *The Review of Economic Studies*, 51:353–368, 1984.

[69] M. Southwell. Beyond the pos. *Business Trends*, July 2, 2002.

[70] P. Steiner. Peak loads and efficient pricing. *Quarterly Journal of Economics*, 71:585–610, 1957.

[71] M. Stiving. Price-endings when prices signal quality. *Management Science*, 46:1617–1629, 2000.

[72] N. Stokey. Intertemporal price discrimination. *Quarterly Journal of Economics*, 93:355–371, 1979.

[73] L. A. Stole. Price discrimination in competitive environments. Working Paper, Graduate School of Business, University of Chicago, 2001.

[74] L. Thomas. Price-production decisions with deterministic demand. *Management Science*, 16:747–750, 1970.

[75] G.T. Thowsen. A dynamic nonstationary inventory problem for a price/quantity setting firm. *Navel Research Logistics Quarterly*, 22:461–476, 1975.

[76] H.R. Varian. Price discrimination. *Handbook of Industrial Organization*, 1:597–654, 1989. R. Schmalensee and R. Willig, eds.

[77] A. Veinott and H. Wagner. Computing optimal (s,s) inventory policies. *Management Science*, 11(5):525–552, 1965.

[78] J. Villas-Boas. Dynamic competition with customer recognition. *RAND Journal of Economics*, 30(4):604–631, 1999.

[79] J. M. Villas-Boas. Price cycles in markets with customer recognition. Working Paper, Haas School of Business, University of California at Berkeley, 2001.

[80] E.J. Warner and R.B. Barsky. The timing and magnitude of retail store markdowns: Evidence from weekends and holidays. *The Quarterly Journal of Economics*, 110:321–352, 1995.

[81] L.R. Weatherford and S. Bodily. A taxonomy and research overview of perishable-asset revenue management: Yield management, overbooking, and pricing. *Operations Research*, 40:831–844, 1992.

[82] C.A. Yano and S.M. Gilbert. Coordinated pricing and production/procurement decisions: A review. *Managing Business Interfaces: Marketing, Engineering and Manufacturing Perspectives*, 2002. A. Chakravarty and J. Eliashberg (Eds.), Kluwer Academic Publishers.

[83] E. Zabel. Monopoly and uncertainty. *The Review of Economic Studies*, 37:205–219, 1970.

[84] W. Zhao and Y.-S. Zheng. Optimal dynamic pricing for perishable assets with nonhomogeneous demand. *Management Science*, 46(3):375 388, 2000.

[85] P. Zipkin. *Foundations of Inventory Management*. McGraw-Hill, 2000. Burr Ridge IL.

Online Appendix:

Citations to other branches of pricing literature (Supplement to Section 1 and 2)

The landscape of pricing methods is wide and varied. Providing an overview of the entire pricing literature even with a glimpse on each topic would be a daunting task in a journal article. Given its applicability in most markets and its increasing adoption in practice, our focus in this survey is on dynamic (intertemporal) pricing in the presence of inventory considerations. Topics that are equally important but are not covered in this review include: (*i*) Optimal single prices with inventory considerations [39] [45]. (*ii*) Static product line pricing, e.g., pricing of products with interdependent demand, such as complementary or substitute products, and additions/deletions of products in the line and subsequent price changes [64]. (*iii*) $1^{st} - 3^{rd}$ degree price discrimination, e.g., customized pricing or quantity discounts [11] [27] [33] [43] [76] [79]. (*iv*) Pricing to signal quality [71]. (*v*) Pricing in the presence of network externalities [19]. (*vi*) Pricing to cover capacity costs (peak load pricing) [70] (*vii*) Promotion pricing (pre-announced temporary price reductions typically used to increase traffic through a store) [60]. (*viii*) Dynamic pricing as a result of collusion [32] [66].

In addition to the NR-I and R-I-M markets, there is also a large body of literature that focuses on market environments where the seller may replenish inventory and demand is dependent over time (R-D-S [4] [17] [44] [68] [72] and R-D-M [2] [16] [20] [37] [38] [61] [65]). The dependence of demand on past sales can include two components: (*i*) when most customers are interested in purchasing few units of the product, often only one, a purchase today means one less customer in the market, and hence, results in one less sale in the future; (*ii*) demand is governed by a *diffusion* process, i.e., future sales are positively correlated with past sales. This would be the case for products where there are strong word-of-mouth effects and uncertainty surrounding new products in the marketplace. Since most of such products are durables with long life cycles, this line of research focuses on long term, or strategic issues of pricing during the lifetime of a product. However, it does not consider supply-side decisions, such as inventory replenishment, and ignores capacity restrictions. Furthermore, we are not aware of any dynamic price optimization decision-support tools that aid companies in making long-term, life-cycle pricing decisions. As a result, we have omitted these papers from our review.

Additional Factors that Influence a Dynamic Pricing Policy (Supplement to Section 2)

Cost of implementing price changes and business rules: Depending on the sales channel, changing prices might be costly, affecting the frequency, the magnitude or the flexibility of price changes. For example, in a bricks-and-mortar store, changing prices would require updating the price tags manually, which is quite costly. In contrast, changing prices on the Internet can be done virtually at no cost. A company may also have business rules or a corporate pricing strategy that place limitations on the prices a firm can set. For example, prices may only be allowed to decrease over time, at most one price reduction is allowed

during a selling season, or there is an upper bound on the maximum price difference between a national and a private brand.

Seasonality of demand: Demand might follow a different pattern over time due to factors such as holidays, seasons, and weekends. For example, movie theaters usually see higher demand in the evenings or on weekends than during weekdays. Seasonal items such as snowblowers or beach towels have peak demand during certain months of the year and low demand otherwise.

External shocks to demand: External events, which are outside the control of a firm, might influence the demand for certain products. For example, an unusually rainy season (a 'local' external event) in a region might increase the demand for umbrellas. On the other hand, a 'global' event such as the September 11, 2001 attacks might decrease the demand for many goods, including travel services.

Cross-elasticities: The demand of a product might depend on the other complement and substitute products and their prices available in the market. For example, generic and private brands (e.g., acetaminophen and Tylenol) of the same item are substitutes, and hence the demand for the private brand would depend not only on the price of the private brand, but also on the price of the generic brand. Similarly, demands for complementary products such as razors and razor blades, cameras and films etc. are also influenced by both products' prices.

The Evolution of Markdown Pricing (Supplement to Section 3)

In the last couple of decades, the variety of goods sold in the market has increased significantly, while the product life cycles have become shorter. This is especially true in fashion retailing, where the greater use of colors and prints, as well as the increasing demand for sportswear (for which there are few accepted guidelines compared to formal clothing) all contribute to increased product diversity and demand uncertainty. For example, the market share of white sheets dropped from about 65% in the mid 1960s to about 16% by 1975 [56]. Similarly fancy (stripes, patterns, etc.) men's shirts and women's sportswear gained increasing market share over the past five decades. Although improved supply chain practices and production technologies helped increase responsiveness, long lead times due to overseas imports and shorter selling seasons result in larger forecasting errors and an inability to change inventory levels in response to demand. As a result, production/inventory decisions have to be made in advance with little information about demand, before the actual selling begins. Given that the inventory levels and the length of the selling season are predetermined, retailers have been increasingly turning to markdowns to clear excess inventory and to balance demand and supply.

According to the National Retail Federation, marked-down goods, which accounted for just 8% of department-store sales three decades ago, now account for over 20% of sales [52]. To better understand the evolution of markdown pricing, several researchers have analyzed sales data and proposed different hypotheses to explain the increase in the depth and the frequency of markdowns.

Analyzing data from 1925 to 1984, Pashigian (1988) observes that the percentage markups as well as the amount of merchandise sold at markdown prices have increased significantly over time. This situation is explained by the fashion hypothesis: both markups and markdowns will increase as uncertainty (fashion) increases. Pashigian defines *fashion* as greater uncertainty facing stores about the future popularity of styles. Analytical models (e.g., [46]) indicate that the percentage markup and the frequency of markdowns will be higher for product groups with higher demand uncertainty (or higher variability in customer valuations). As products change more frequently, uncertainty also increases, explaining partly the higher markdowns seen in women's or junior's fahion clothing compared to other product groups, e.g., men's clothing. These predictions are also supported by the empirical analysis of Pashigian and Bowen (1991), who observe that (1) fancy shirts have a higher probability of selling at markdown followed by other solid and then white shirts, and (2) the average percentage markdown and the probability of being sold on sale are higher on shirts with higher regular prices.

Pashigian and Bowen observe two patterns in practice, which cannot be fully explained by the fashion hypothesis. First, there is a large number of markdowns in the pre- rather than post-Christmas season. Similarly, the percentage of shirts sold on sale at the beginning of each selling season is unexpectedly high. Second, more price drops occur on weekends and holidays, when a larger number of customers with possibly less elastic demand curves (since many of them are working people with higher household incomes) visit the stores. By studying prices of eight goods at seventeen retail stores (collected in Ann Arbor, Michigan over a four-month period from November 1 to February 28), Warner and Barsky (1995) also observe the "weekend effect" – prices fall as the weekend approaches and rise on Monday. These observations are also in contradiction with the peakload hypothesis, which predicts that prices will be higher in periods of high demand.

Warner and Barsky (1995) explain the lower prices on days characterized by high intensity of shopping activity (e.g., weekends, or pre-Christmas days) via the "thick market" hypothesis. They claim that on such days, consumers usually purchase multiple items and hence the search costs per item can be lower, leading to better informed customers. Therefore, retailers perceive their demand to be more elastic, and respond with lower prices. Focusing mainly on temporary markdowns and regular (rather than high-fashion) items, they make the following observations: (1) prices oscillate between two or three benchmark levels; (2) sales before Christmas tend to be more frequent but shorter in duration than sales after Christmas; (3) the largest markdowns occur after Christmas and are premanent markdowns; (4) temporary price reductions occur usually on weekends.