Computer Aided Design of Industrial Logistics Systems

Marc Goetschalckx, Ph.D.
George Nemhauser, Ph.D.
Michael H. Cole
Ru-Pei Wei
Koray Dogan
Xiaowei Zhang

Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0205
(404) 894-2317

Version 1.1, August 1, 1994.

DRAFT

This research has been supported in part by the member companies of the Material Handling Research Center of the Georgia Institute of Technology.

Abstract

Recent geopolitical changes, the continuing development of truly global corporations, and the increasing importance of make-to-order manufacturing and inventory reductions in distribution systems have placed further emphasis on the growing role of logistics in the current business environment. One of the needs most often identified by companies is a tool to rapidly evaluate and design different logistics philosophies and configurations. This tool must be comprehensive, i.e. include all relevant costs and constraints, integrated, i.e. cover the logistics from supplier to final customer, systematic, i.e. provide high quality designs, and support easy manipulation and sensitivity analysis.

In this manuscript we report our continuing research in developing a comprehensive model for the design of integrated logistics systems and its associated solution techniques. In particular, we will focus on SMILE, a mixed integer programming algorithm to solve this model to optimality. Simultaneously, we also continue to develop the CIMPEL (Computer Integrated Modeling and Planning Environment for Logistics) tool, which allows easy user interaction, sensitivity analysis, and graphical displays. We will also comment on some recent case studies executed with this model, solution algorithm, and tool.

1. Introduction

Logistics is concerned with the movement, storage, and control of material from the suppliers, through the manufacturing and distribution facilities, all the way to the customer. As such it is one of the central functions of any production or manufacturing organization. Logistics encompasses many other disciplines such as facilities design, production and inventory control, and distribution and transportation. Because of the many interrelated subfunctions it is hard to measure the true cost of logistics and to design effective logistics systems.

In recent years several factors have contributed to the rapid changes in the area of logistics. Many corporations have developed a truly global view, where purchasing, production, and distribution are located in the most economical place anywhere in the world. Several major geopolitical changes occurred which opened new markets in Russia, China, and Eastern Europe. Increasing pressure to achieve the greater efficiency necessary for corporate survival and the associated business reengineering has led to a large number of reorganizations, acquisitions, and divestitures. Finally, lean and rapid response manufacturing generated more frequent and smaller production quantities with vastly reduced lead times.

The result of all these changes is that management has recognized distribution and logistics as a competitive tool to increase service and decrease cost. The above factors have also created the need for a continuing, integrated review and redesign of the logistics systems used by the corporation to adapt to the constantly changing business environment.

Traditionally, the design of strategic logistics and distribution systems has followed the four phases shown in the Figure 1. First, the customer service requirements are established to support the organization's business goals and strategy and marketing plans. Service levels may take the form of percentage delivered out of stock. Second, the materials deployment strategy is developed, which answers such questions as make-or-buy and in which manufacturing plant the goods will be produced. Third, a distribution network strategy is developed. Typical decisions are the number and location of distribution centers. Finally, a transportation strategy is established which allocates customers to distribution centers and selects transportation modes and carriers. It should be observed that we do not consider the operational decisions such as which truck and driver will serve a set of customers and in what order. Such problems belong to the class of vehicle routing problems and an recent overview is given in Golden and Assad (1989). Early papers on the network design problem were written by Geoffrion and Graves (1974) and Geoffrion et al (1974). An overview of such models is provided in Aikens (1985). Simultaneously, a large body of literature has appeared on inventory decisions. An overview is given in Silver and Peterson (1985).

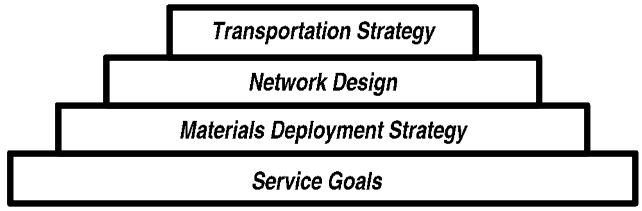

Figure 1. Traditional Logistics Design Phases

But, researchers realized that focussing exclusively on one of the above decision levels leads to suboptimization of the overall design problem. Several researchers integrated two levels of the above pyramid and extended the early models. Blumenfeld et all (1985), Bookbinder and Reece (1988), and Larson (1988) analyzed the tradeoffs between transportation and inventory costs.

The gains in productivity derived from integrated logistics systems can be substantial. Kearney (1984) reports that industrial corporations saved more than 12 % when they went to an integrated approach of design and operation of distribution systems. Perl and Sirisponsilp (1988) attempted for the first time to build a truly comprehensive model for the design of strategic distribution systems. They identified the interrelating factors in the areas of transportation, inventory, and location and proposed a model. No solution methods or computational results were reported.

The strategic goal of this research is thus to develop a basic understanding, design methodology, and decision making theory for integrated logistics systems. A schematic illustration of this process is given in the Figure 2. Here materials deployment, network design, and transportation strategy are all combined into a single integrated logistics model. Such models tend to be very large and very complex and their optimal solution is mathematically difficult. In previous research we developed a heuristic solution algorithm, called LAIR, see Song et all (1990). In this research we have developed an optimal solution based on mixed integer programming algorithms, denoted by SMILE.

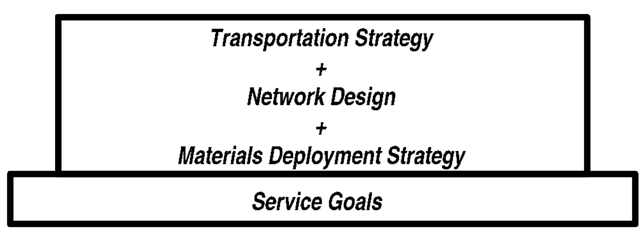

Figure 2. Modern Integrated Logistics Design

2. Integrated Logistics Model

Conceptually, the various logistics activities often can be viewed as an abstract network of nodes and arcs. The nodes represent facilities, points where product flows are created, processed, stored, and consumed (suppliers, plants, distribution centers, warehouses, customers). The arcs which are also called transportation channels, represent the flows or movements of products between the various facilities, or nodes. There may be several transportation channels between a pair of facilities to represent alternate form of transportation means, different shipping frequencies, and different cost structures.

The model as presented in this paper considers all material movement and storage from the moment the material leaves the manufacturing plant until it arrives at the customer. The model can handle multicommodity (i.e. more than one product group), multiecholon (i.e. zero, one, or more distribution centers between plant and customer), multichannel (i.e. alternative transportation channels between an origin and destination facility) problems. We have restricted ourselves to static or one period deterministic models (i.e. best, worst or average scenarios or future time period scenarios have to be created independently). Single sourcing of customers can be incorporated. The model incorporates production costs. The model has fixed costs associated with establishing or closing any facility. In addition, the model allows one step assembly operations in each of the distribution centers.

The remainder of this section lists basic assumptions, costs, and decisions associated with the various components of a strategic logistics system.

2.1. Model Components

2.1.1. Products

A set of products is given as input. All the products have the same basic unit load, e.g., pallet, truck load, or car load. Each product has value, weight, and volume per unit load. The value, weight, and volume per unit load of a product are constant throughout the logistics network. A product may be a combination of several different products; the model allows one step assembly operations in each of the warehouses. The part structure or bill of materials is limited to at most one level. That is, a product assembled from various component products cannot itself be used as a component product. The number of component products required to assemble a final product is prespecified. Total assembly cost is proportional to the number of final products assembled.

2.1.2. Plants

A set of potential production plants is given as input. Each plant has a location, a maximum production resource capacity, a fixed facility cost, and produces at a steady rate. A constant proportional cost is incurred for each unit load of product produced at a plant. The production cost per unit may differ among plants and products. Some plants may not produce some products. A constant proportional resource is consumed for each unit load of product produced at a plant. The resource requirement per unit may differ among plants and products. In addition, finished goods kept at a plant incur inventory holding costs according to a given holding cost rate. The production decisions are which plants to open and how much to produce for each product at each plant.

2.1.3. Warehousing

A set of potential warehouses is given. Any number of warehouses between plants and customers is possible, i.e., the system can be multiechelon. Associated with each potential warehouse are a location, maximum handling and storage capacities, and fixed and variable facility costs. All the different products are handled and stored in the same basic unit load; e.g., palletes or truck loads, and all the product flows, warehouse capacities, warehouse storage costs, and warehouse handling costs are expressed in terms of the basic unit load.

Since the problem is strategic and computing the cycle inventory in a multicommodity inventory system requires determining the phasing of inventory replenishments, reordering cycle time, and lot sizes (Hall, 1988), the cycle inventory will be treated with approximations. It is assumed that each warehouse has many uncorrelated, unscheduled, and independent demands, so the product outflow of each warehouse is approaximately constant per fundamental time period and the product inflow is replenished simultaneously. The amount of safety stock is proportional to the throughput to model the inventory philosophy that a user determined number of periods of demands (safety stock factor) are held for safety stock. Each warehouse may perform assembly operations to combine different products into one another. There is no capacity limit on assembly operations. All assembly operations are performed instantaneously before products leave the warehouse. Warehousing decisions comprise which warehouses to open and the amount of product flow through each warehouse.

2.1.4. Transportation

Transportation can be divided into trunking and local delivery operations. Trunking comprises direct shipments from plants to warehouses, and from higher echelon (central) warehouses to lower echelon (local) warehouses. Local delivery comprises shipments from plants (direct shipment) or warehouses to customers. Local delivery options include direct shipment, peddling, and zoned delivery. The cost structure differs for each option. Since the problem is strategic, solving this strategic problem does not entail designing vehicle routes for local warehouses, which is a tactical or operational concern. Local delivery costs will be treated with approximations or as given. Each transportation channel has a shipping interval, deterministic travel time, travel distance, maximum allowable number of carriers, minimum number of required carriers, carrier weight capacity, carrier volume capacity, a cost per carrier, and a cost per unit load shipped. Pipeline inventories incur holding costs according to a given holding cost rate. Transportation decisions include allocation of customers to facilities, channel selection and the amount of each shipment.

2.1.5. Customers

A set of customers is given as input. Customers have deterministic demands for each of the products. Customer demands are steady over time, and do not depend on the level of service provided. Each customer is served every day. Some customers may require being served from a single facility, which is called the single sourcing requirement. Otherwise, customer orders may be split over different facilities and different transportation channels. The customer service requirements considered in current model are customer demand satisfaction, warehouse safety stock, single sourcing, and a maximum travel distance limitation between a customer and the warehouse that serves it.

The distribution centers have fixed costs, variable costs for handling different commodities and also variable storage costs for storing commodities. Transportation channels are capacitated both by weight and volume. Transportation costs are in terms of carrier containers such as trucks, railroad boxcars etc.., and/or proportional to the amount of material shipped, and/or proportional to the amount of material shipper per distance unit. The model also accounts for all inventory costs throughout the system. The amount of inventory in distribution centers is proportional to the throughput to model the inventory philosophy that a user determined number of periods of demands are held in inventory.

2.2. Verbal Formulation

Minimize total cost =

warehouse inventory cost	(Z1)
+ warehouse assembly cost	(Z2)
+ warehouse facility cost	(Z3)
+ plant production cost	(Z4)
+ plant inventory cost	(Z5)
+ plant facility cost	(Z6)
+ shipment cost	(Z7)
+ transportation inventory cost	(Z8)

Subject to:

flow conservation	(FC)
general flow conservation	(GF)
warehouse flow capacity	(TH, IF)

warehouse storage capacity	(ST, IS)
one warehouse type per site	(DT)
plant production capacity	(SP)
minimum/maximum number of carriers	(IC, XC)
carrier weight/volume capacity	(WE, VO)
ship to/from open warehouse only	(TD,FD)
ship from open plant only	(FP)
customer single sourcing	(SS1, 2, 3, 4)
max customer to warehouse distance	(MD)
customer demand satisfaction	(DM)
restrictions on variable values	

2.3. Notation

For consistency, all time-based parameters and variables are described in terms of days. Another fundamental time period can be substituted everywhere for the days.

2.3.1. Sets and Indexes

B : set of manufacturing facilities or plants, indexed by i

C : set of customers, indexed by k

CS : set of customers with single sourcing requirements ($CS \subseteq C$), indexed by k

D : set of warehouses or depots, indexed by j

L : set of warehouse types or sizes (e.g., small, medium, large, public), indexed by l

M : set of different transportation channels between a pair of facilities, indexed by m

P : set of products, indexed by p

PN : set of products not involved in assembly operations ($PN \subseteq P$), indexed by p

PC : set of all the component products of all the assembly operations ($PC \subseteq P$), indexed by p

PF : set of all final products of all the assembly operations ($PF \subseteq P$), indexed by f

PFp : set of all final products that the component product p may assemble ($PFp \subseteq PF$), indexed by f

2.3.2. Parameters

$PCap_i$: production capacity of plant i (resource units per day)

PFC_i : fixed cost of plant i ($ per day)

PPC_{ip} : production cost rate of plant i to produce one unit load of product p ($ per unit load)

PRU_{ip} : production resource units required for plant i to produce one unit load of product p (resource units per unit load)

WFC_{jl} : fixed cost of type l warehouse at site j ($ per day)

WHC_{jl} : handling cost of type l warehouse at site j ($ per unit load flow)

$WHCap_{jl}$: maximum handling capacity of type l warehouse at site j (unit loads per day)

WSC_{jl} : storage capacity cost of type l warehouse at site j ($ per unit load storage capacity per day)

$WSCap_{jl}$: maximum storage capacity of type l warehouse at site j (unit loads)

ssf_{jp} : safety stock factor of product p at warehouse j

AC_f : cost to assemble one unit load of final product f ($ per unit load)

AQt_{pf} : number of unit loads of component product p required to assemble one unit load of final product f

DM_{kp} : demand mean of customer k for product p (unit loads per day)

$MaxDistance_k$: maximum allowable distance from customer k to its server warehouse (miles)

$Distance_{jkm}$: travel distance on transportation channel jkm (miles)

T_{ijm} : transport time on transportation channel ijm (days)

R_{ijm} : time between shipments on transportation channel ijm, i.e., order interval (days)

TCC_{ijm} : shipment cost per carrier on transportation channel ijm ($ per carrier shipment)

TUC_{ijm} : shipment cost per unit load on transportation channel ijm ($ per unit load shipment)

$MinC_{ijm}$: minimum number of carriers required per order interval on transportation channel ijm (carriers per order interval)

$MaxC_{ijm}$: maximum allowable number of carriers per order interval on transportation channel ijm (carriers per order interval)

$CWCap_{ijm}$: carrier weight capacity of transportation channel ijm (weight units per carrier)

$CVCap_{ijm}$: carrier volume capacity of transportation channel ijm (volume units per carrier)

Val_p : value of a unit load of product p ($ per unit load)

Wt_p : weight of a unit load of product p (weight units per unit load)

Vol_p : volume of a unit load of product p (volume units per unit load)

r : inventory carrying cost rate ($ per $ per day)

2.3.3. Decision Variables

x_{ijmp} : average amount of product p shipped over transportation channel ijm (unit loads per day)

v_{jlp} : average amount of product p flow through warehouse of type l at site j (unit loads per day)

u_{jlp} : maximum amount of product p stored at warehouse of type l at site j (unit loads)

w_{ijm} : number of carriers used in one shipment from source facility i to destination facility j via transportation mode m

b_i : (0,1) 1 if plant is opened at site i, 0 otherwise

y_{jl} : (0,1) 1 if type l warehouse is opened at site j, 0 otherwise

z_{ijm} : (0,1) 1 if there is an open channel from source facility i to destination facility j via transportation mode m, 0 otherwise

q_{jk} : (0,1) 1 if customer k is served from facility j, 0 otherwise

2.4. Detailed Development

2.4.1. Warehousing

It is assumed that every R days, each warehouse places an order for a product. The sketch below shows an idealized single product inventory cycle.

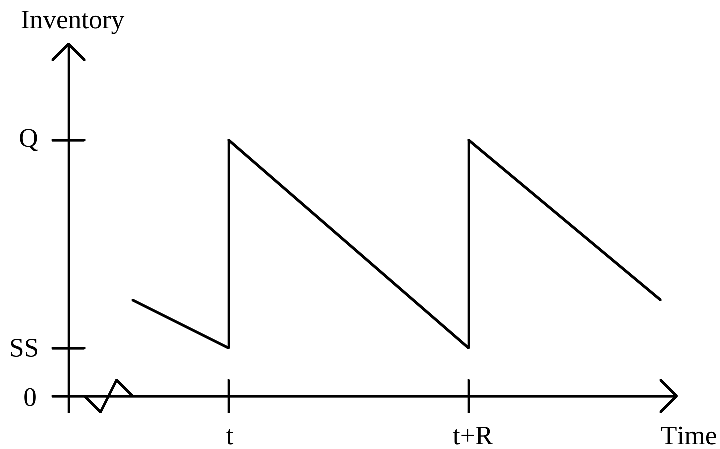

Figure 3. Idealized single product inventory cycle

Warehouse j placed an order and after the lead-time the order is received at time t. The inventory reach its highest level Q (Q = SS + demand rate × R) at time t. The cycle repeats after the next order is placed. The next order will be received at time t+R.

The warehouse inventory cost is simply the product of the inventory holding cost rate and the value of the average inventory. The mathematical form is developed below.

R = order interval

r = inventory holding cost rate

D_{jp} = demand rate for product *p* at warehouse *j*

$$D_{jp} = \sum_{i \in B \cup D} \sum_{m \in M} x_{ijmp} = \sum_{k \in C \cup D} \sum_{m \in M} x_{jkmp}$$

For convenience of modeling, it's assumed that the products are replenished simultaneously.

Q_{jp} = expected maximum inventory of product *p* at warehouse *j*

SS_{jp} = safety stock of product *p* at warehouse *j*

$$Q_{jp} = SS_{jp} + \sum_{i \in B \cup D} \sum_{m \in M} R_{ijm}\, x_{ijmp}$$

Safety stock is the product flow times a preset safety stock factor, e.g. three days of product flow.

$$SS_{jp} = \sum_{i \in B \cup D} \sum_{m \in M} ssf_{jp} \; x_{ijmp}$$

$$Q_{jp} = \sum_{i \in B \cup D} \sum_{m \in M} ssf_{jp} \; x_{ijmp} + \sum_{i \in B \cup D} \sum_{m \in M} R_{ijm} \; x_{ijmp}$$

AI_{jp} = average inventory of product *p* at warehouse *j*

$$AI_{jp} = \sum_{i \in B \cup D} \sum_{m \in M} ssf_{jp} \; x_{ijmp} + \left(\frac{1}{2}\right) \sum_{i \in B \cup D} \sum_{m \in M} R_{ijm} \; x_{ijmp}$$

Thus, the warehouse inventory cost for warehouse j is

$$\sum_{p \in P} r \; Val_p \left[\sum_{i \in B \cup D} \sum_{m \in M} ssf_{jp} \; x_{ijmp} + \left(\frac{1}{2}\right) \sum_{i \in B \cup D} \sum_{m \in M} R_{ijm} \; x_{ijmp} \right]$$

and the total system wide warehouse inventory cost is

$$\sum_{j \in D} \sum_{p \in P} r \; Val_p \left[\sum_{i \in B \cup D} \sum_{m \in M} ssf_{jp} \; x_{ijmp} + \left(\frac{1}{2}\right) \sum_{i \in B \cup D} \sum_{m \in M} R_{ijm} \; x_{ijmp} \right] \tag{Z1}$$

This section develops the model of the assembly operations at warehouses. Moiseenko (1984) presents a generalized multicommodity network flow model which allows conversion of different commodities. Still, commodity conversion can not model assembly operations; the product assembly is a more general problem.

For those products that are not involved in any assembly operations, constraint (FC) enforces conservation of flow at each warehouse.

$$\sum_{i \in B \cup D} \sum_{m \in M} x_{ijmp} = \sum_{k \in C \cup D} \sum_{m \in M} x_{jkmp} \qquad j \in D, \; p \in PN \tag{FC}$$

However, flow conservation does not hold true for assembly operations. Consider the following example where two units of product A and three units of product B are combined to form one unit of product C; two units of product B and one unit of product D are combined to form one unit of product E. The typical assembly equations for the operations are:

$$2\,A + 3\,B \rightarrow C$$

$$2\,B + 1\,D \rightarrow E$$

In this case, the flow conservation condition must be replaced by some more general constraints. Let x denote the input flows and y denote the output flows. The general flow conservation equations for the warehouse assembly operations are

$$x_A - y_A = 2\,(y_C - x_C)$$

$$x_B - y_B = 3\,(y_C - x_C) + 2\,(y_E - x_E)$$

$$x_D - y_D = (y_E - x_E)$$

The general flow conservation conditions for warehouse assembly operations are as follows:

$$\sum_{i \in B \cup D} \sum_{m \in M} x_{ijmp} - \sum_{k \in C \cup D} \sum_{m \in M} x_{jkmp} = \sum_{f \in PF_p} AQt_{pf} \left(\sum_{k \in C \cup D} \sum_{m \in M} x_{jkmf} - \sum_{i \in B \cup D} \sum_{m \in M} x_{ijmf} \right) \quad j \in D,\ p \in PC \qquad \text{(GF)}$$

The total warehouse assembly cost is

$$\sum_{j \in D} \sum_{f \in PF} AC_f \left(\sum_{k \in C \cup D} \sum_{m \in M} x_{jkmf} - \sum_{i \in B \cup D} \sum_{m \in M} x_{ijmf} \right) \qquad \text{(Z2)}$$

The facility cost of each warehouse includes a cost proportional to storage capacity, a cost proportional to the amount shipped through the warehouse, and a fixed cost. Each of these costs is defined in terms of a warehouse site and a warehouse type. In general, each potential warehouse site can be occupied by one of several types of warehouses. For example, small, medium, and large may be three possible types of warehouse. The notion of multiple warehouse types, each with a linear cost structure, allows a nonlinear (but piecewise linear) cost structure for each potential warehouse site.

The fixed cost of a warehouse accounts for overhead, capital costs, and such that are not considered to be proportional to storage capacity or throughput.

The various products are assumed to be handled and stored in the same unit loads, e.g., pallets, boxes, or trucks. All the product flows, warehouse capacities, warehouse storage costs, and warehouse handling costs are expressed in terms of the same unit loads.

The required storage capacity of a warehouse is determined by Q_{jp}, the previously derived maximum amount of inventory of product *p* stored at the warehouse.

The maximum amount of a product stored at a warehouse, u_{jlp}, is defined by the constraints (ST) and (IS). Constraint (ST) enforces the required warehouse storage capacity. Constraint (IS) states that the maximum amount stored at a warehouse is equal to the expected maximum inventory at a warehouse.

$$\sum_{p\in P} u_{jlp} \leq WSCap_{jl}\, y_{jl} \qquad j \in D,\ l \in L \qquad \text{(ST)}$$

$$\sum_{l\in L} u_{jlp} = \sum_{i\in B\cup D}\sum_{m\in M} ssf_{jp}\, x_{ijmp} + \sum_{i\in B\cup D}\sum_{m\in M} R_{ijm}\, x_{ijmp} \quad j \in D,\ p \in P \qquad \text{(IS)}$$

The amount shipped through a warehouse, vjlp, is defined by the constraints (TH) and (IF). Constraint (TH) enforces warehouse handling capacity. Constraint (IF) states that the flow through a warehouse is equal to the flow into a warehouse.

$$\sum_{p\in P} v_{jlp} \leq WHCap_{jl}\, y_{jl} \qquad j \in D,\ l \in L \qquad \text{(TH)}$$

$$\sum_{l\in L} v_{jlp} = \sum_{i\in B\cup D}\sum_{m\in M} x_{ijmp} \qquad j \in D,\ p \in P \qquad \text{(IF)}$$

Thus, the total warehousing facility cost is

$$\sum_{j\in D}\sum_{l\in L}\left(WFC_{jl}\, y_{jl} + WSC_{jl}\sum_{p\in P} u_{jlp} + WHC_{jl}\sum_{p\in P} v_{jlp} \right) \qquad \text{(Z3)}$$

Constraint (DT) requires that at most one type of warehouse be opened at a site.

$$\sum_{l\in L} y_{jl} \leq 1 \qquad j \in D \qquad \text{(DT)}$$

2.4.2. Production

Plant costs include production costs, inventory costs, and fixed costs.

Plant production costs is proportional to each unit load of product produced at a plant, which is expressed in the following:

$$\sum_{i\in B}\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P} PPC_{ip}\, x_{ijmp} \qquad \text{(Z4)}$$

Plant inventory costs:

$$\left(\frac{1}{2}\right)\sum_{i\in B}\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P} r\ \mathrm{Val}_p\ \mathrm{R}_{ijm}\ \mathrm{x}_{ijmp} \tag{Z5}$$

Plant fixed costs:

$$\sum_{i\in B} \mathrm{PFC}_i \mathrm{b}_i \tag{Z6}$$

Constraint (SP) prevents a plant from shipping product at a rate greater than its production capacity.

$$\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P} \mathrm{PRU}_{ip}\ \mathrm{x}_{ijmp} \ \le\ \mathrm{PCap}_i \qquad \mathrm{i}\in \mathrm{B} \tag{SP}$$

2.4.3. Transportation

Shipment costs comprise a variable cost proportional to the number of carriers used, the amount shipped, or both.

$$\sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M}\left(\frac{\mathrm{TCC}_{ijm}}{\mathrm{R}_{ijm}}\right)\mathrm{w}_{ijm} + \sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P} \mathrm{TUC}_{ijm}\mathrm{x}_{ijmp} \tag{Z7}$$

Transportation inventory costs are

$$\sum_{\mathrm{i}\in \mathrm{B}\cup \mathrm{D}}\sum_{\mathrm{j}\in \mathrm{C}\cup \mathrm{D}}\sum_{\mathrm{m}\in \mathrm{M}}\sum_{\mathrm{p}\in \mathrm{P}} \mathrm{r\ Val_p\ T_{ijm}x_{ijmp}} \tag{Z8}$$

Each transportation channel has a required minimum number of carriers used (maybe zero), which is the enforced by constraint (IC),

$$\mathrm{w_{ijm}} \ \ge\ \mathrm{MinC_{ijm}z_{ijm}} \qquad \mathrm{i}\in \mathrm{B}\cup \mathrm{D},\ \mathrm{j}\in \mathrm{C}\cup \mathrm{D},\ \mathrm{m}\in \mathrm{M} \tag{IC}$$

and a limited maximum allowable number of carriers used (maybe infinity), enforced by constraint (XC).

$$\mathrm{w_{ijm}} \ \le\ \mathrm{MaxC_{ijm}z_{ijm}} \qquad \mathrm{i}\in \mathrm{B}\cup \mathrm{D},\ \mathrm{j}\in \mathrm{C}\cup \mathrm{D},\ \mathrm{m}\in \mathrm{M} \tag{XC}$$

Each carrier has a weight capacity, enforced by constraint (WE),

$$\sum_{p\in P} \mathrm{R}_{ijm}\mathrm{Wt}_p\mathrm{x}_{ijmp} \ \le\ \mathrm{CWCap}_{ijm}\mathrm{w}_{ijm} \qquad \mathrm{i}\in \mathrm{B}\cup \mathrm{D},\ \mathrm{j}\in \mathrm{C}\cup \mathrm{D},\ \mathrm{m}\in \mathrm{M} \tag{WE}$$

and a volume capacity, enforced by constraint (VO).

$$\sum_{p \in P} \mathrm{R}_{ijm} \mathrm{Vol}_{p} \mathrm{x}_{ijmp} \leq \mathrm{CVCap}_{ijm} \mathrm{w}_{ijm} \qquad \mathrm{i} \in \mathrm{B} \cup \mathrm{D},\ \mathrm{j} \in \mathrm{C} \cup \mathrm{D},\ \mathrm{m} \in \mathrm{M} \qquad \text{(VO)}$$

Constraints (TD) and (FD) require that transportation channels to and from a potential depot site are usable only if a depot is actually opened at that site.

$$\mathrm{z}_{ijm} \leq \sum_{l \in L} \mathrm{y}_{jl} \qquad \mathrm{i} \in \mathrm{B} \cup \mathrm{D},\ \mathrm{j} \in \mathrm{D},\ \mathrm{m} \in \mathrm{M} \qquad \text{(TD)}$$

$$\mathrm{z}_{jkm} \leq \sum_{l \in L} \mathrm{y}_{jl} \qquad \mathrm{j} \in \mathrm{D},\ \mathrm{k} \in \mathrm{C} \cup \mathrm{D},\ \mathrm{m} \in \mathrm{M} \qquad \text{(FD)}$$

Constraint (FP) requires that transportation channels from a plant are usable only if a plant is open.

$$\mathrm{z}_{ijm} \leq \mathrm{b}_{i} \qquad i \in B,\ j \in C \cup D,\ m \in M \qquad \text{(FP)}$$

Constraints (SS1) enforces that the customers requiring single sourcing be served by exactly one source.

$$\sum_{j \in B \cup D} \mathrm{q}_{jk} = 1 \qquad k \in CS \qquad \text{(SS1)}$$

Constraints (SS2 and SS3) require that a customer can only be served from an open plant or an open warehouse.

$$\mathrm{q}_{ik} \leq \mathrm{b}_{i} \qquad i \in B,\ k \in CS \qquad \text{(SS2)}$$

$$\mathrm{q}_{jk} \leq \sum_{l \in L} \mathrm{y}_{jl} \qquad j \in D,\ k \in CS \qquad \text{(SS3)}$$

Constraint (SS4) requires that transportation channels from a source to a customer are usable only if the customer is served by that source.

$$\mathrm{z}_{jkm} \leq \mathrm{q}_{jk} \qquad j \in B \cup D,\ k \in CS,\ m \in M \qquad \text{(SS4)}$$

Constraint (MD) requires that each customer be within a specified maximum travel distance from the warehouse or warehouse that serves it.

$$\sum_{j \in B \cup D} \sum_{m \in M} \mathrm{Distance}_{jkm} \mathrm{z}_{jkm} \leq \mathrm{MaxDistance}_{k} \qquad \mathrm{k} \in \mathrm{C} \qquad \text{(MD)}$$

2.4.4. Customers

Constraint (DM) ensures demand satisfaction.

$$\sum_{j \in B \cup D} \sum_{m \in M} x_{jkmp} = DM_{kp} \qquad k \in C,\ p \in P \qquad \text{(DM)}$$

2.4.5. Decision Variable Values

The values obtained by the decision variables are constrained as follows.

$$v_{jlp} \geq 0 \qquad j \in D, l \in L, p \in P$$

$$u_{jlp} \geq 0 \qquad j \in D, l \in L, p \in P$$

$$x_{ijmp} \geq 0 \qquad i \in B \cup D,\ j \in C \cup D, m \in M, p \in P$$

$$w_{ijm} \geq 0 \qquad i \in B \cup D,\ j \in C \cup D, m \in M$$

$$b_i \in \{0,1\} \qquad i \in B$$

$$y_{jl} \in \{0,1\} \qquad j \in D,\ l \in L$$

$$z_{ijm} \in \{0,1\} \qquad i \in B \cup D,\ j \in C \cup D, m \in M$$

$$q_{jk} \in \{0,1\} \qquad j \in D,\ k \in CS$$

2.5. Mathematical Formulation

Minimize

$$\sum_{j \in D} \sum_{p \in P} r\, Val_p \left[\sum_{i \in B \cup D} \sum_{m \in M} ssf_{jp}\ x_{ijmp} + \left(\frac{1}{2}\right) \sum_{i \in B \cup D} \sum_{m \in M} R_{ijm}\ x_{ijmp} \right] \qquad \text{(Z1)}$$

$$\sum_{j \in D} \sum_{f \in PF} AC_f \left(\sum_{k \in C \cup D} \sum_{m \in M} x_{jkmf} - \sum_{i \in B \cup D} \sum_{m \in M} x_{ijmf} \right) \qquad \text{(Z2)}$$

$$\sum_{j \in D} \sum_{l \in L} \left(WFC_{jl}\, y_{jl} + WSC_{jl} \sum_{p \in P} u_{jlp} + WHC_{jl} \sum_{p \in P} v_{jlp} \right) \qquad \text{(Z3)}$$

$$\sum_{i \in B} \sum_{j \in C \cup D} \sum_{m \in M} \sum_{p \in P} PPC_{ip} x_{ijmp} \qquad \text{(Z4)}$$

$$\left(\frac{1}{2}\right) \sum_{i \in B} \sum_{j \in C \cup D} \sum_{m \in M} \sum_{p \in P} r\, Val_p\ R_{ijm}\ x_{ijmp} \qquad \text{(Z5)}$$

$$\sum_{i \in B} PFC_i b_i \qquad \text{(Z6)}$$

$$\sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M}\left(\frac{TCC_{ijm}}{R_{ijm}}\right)w_{ijm}+\sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P}TUC_{ijm}x_{ijmp} \tag{Z7}$$

$$\sum_{i\in B\cup D}\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P} r\,Val_p\,T_{ijm}x_{ijmp} \tag{Z8}$$

Subject to:

$$\sum_{i\in B\cup D}\sum_{m\in M}x_{ijmp}=\sum_{k\in C\cup D}\sum_{m\in M}x_{jkmp} \qquad j\in D,\ p\in PN \tag{FC}$$

$$\sum_{i\in B\cup D}\sum_{m\in M}x_{ijmp}-\sum_{k\in C\cup D}\sum_{m\in M}x_{jkmp}=\sum_{f\in PF_p}AQt_{pf}\left(\sum_{k\in C\cup D}\sum_{m\in M}x_{jkmf}-\sum_{i\in B\cup D}\sum_{m\in M}x_{ijmf}\right) \qquad j\in D,\ p\in PC \tag{GF}$$

$$\sum_{p\in P}v_{jlp}\le WHCap_{jl}\,y_{jl} \qquad j\in D,\ l\in L \tag{TH}$$

$$\sum_{l\in L}v_{jlp}=\sum_{i\in B\cup D}\sum_{m\in M}x_{ijmp} \qquad j\in D,\ p\in P \tag{IF}$$

$$\sum_{p\in P}u_{jlp}\le WSCap_{jl}\,y_{jl} \qquad j\in D,\ l\in L \tag{ST}$$

$$\sum_{l\in L}u_{jlp}=\sum_{i\in B\cup D}\sum_{m\in M}ssf_{jp}\,x_{ijmp}+\sum_{i\in B\cup D}\sum_{m\in M}R_{ijm}\,x_{ijmp} \qquad j\in D,\ p\in P \tag{IS}$$

$$\sum_{l\in L}u_{jlp}=\sum_{i\in B\cup D}\sum_{m\in M}ssf_{jp}\,x_{ijmp}+\sum_{i\in B\cup D}\sum_{m\in M}R_{ijm}\,x_{ijmp} \qquad j\in D,\ p\in P \tag{IS}$$

$$\sum_{l\in L}y_{jl}\le 1 \qquad j\in D \tag{DT}$$

$$\sum_{j\in C\cup D}\sum_{m\in M}\sum_{p\in P}PRU_{ip}\,x_{ijmp}\le PCap_i \qquad i\in B \tag{SP}$$

$$w_{ijm}\ge MinC_{ijm}z_{ijm} \qquad i\in B\cup D,\ j\in C\cup D,\ m\in M \tag{IC}$$

$$w_{ijm}\le MaxC_{ijm}z_{ijm} \qquad i\in B\cup D,\ j\in C\cup D,\ m\in M \tag{XC}$$

$$\sum_{p\in P}R_{ijm}Wt_p x_{ijmp}\le CWCap_{ijm}w_{ijm} \qquad i\in B\cup D,\ j\in C\cup D,\ m\in M \tag{WE}$$

$$\sum_{p\in P}R_{ijm}Vol_p x_{ijmp}\le CVCap_{ijm}w_{ijm} \qquad i\in B\cup D,\ j\in C\cup D,\ m\in M \tag{VO}$$

$$z_{ijm} \le \sum_{l\in L} y_{jl} \qquad i \in B \cup D,\ j \in D,\ m \in M \qquad \text{(TD)}$$

$$z_{jkm} \le \sum_{l\in L} y_{jl} \qquad j \in D,\ k \in C \cup D,\ m \in M \qquad \text{(FD)}$$

$$z_{ijm} \le b_i \qquad i \in B,\ j \in C \cup D,\ m \in M \qquad \text{(FP)}$$

$$\sum_{j\in B\cup D} q_{jk} = 1 \qquad k \in CS \qquad \text{(SS1)}$$

$$q_{ik} \le b_i \qquad i \in B,\ k \in CS \qquad \text{(SS2)}$$

$$q_{jk} \le \sum_{l\in L} y_{jl} \qquad j \in D,\ k \in CS \qquad \text{(SS3)}$$

$$z_{jkm} \le q_{jk} \qquad j \in B \cup D,\ k \in CS,\ m \in M \qquad \text{(SS4)}$$

$$\sum_{j\in B\cup D} \sum_{m\in M} \text{Distance}_{jkm} z_{jkm} \le \text{MaxDistance}_k \qquad k \in C \qquad \text{(MD)}$$

$$\sum_{j\in B\cup D} \sum_{m\in M} x_{jkmp} = DM_{kp} \qquad k \in C,\ p \in P \qquad \text{(DM)}$$

$$v_{jlp} \ge 0 \qquad j \in D, l \in L, p \in P$$

$$u_{jlp} \ge 0 \qquad j \in D, l \in L, p \in P$$

$$x_{ijmp} \ge 0 \qquad i \in B \cup D, j \in C \cup D, m \in M, p \in P$$

$$w_{ijm} \ge 0 \qquad i \in B \cup D, j \in C \cup D, m \in M$$

$$b_i \in \{0,1\} \qquad i \in B$$

$$y_{jl} \in \{0,1\} \qquad j \in D,\ l \in L$$

$$z_{ijm} \in \{0,1\} \qquad i \in B \cup D, j \in C \cup D, m \in M$$

$$q_{jk} \in \{0,1\} \qquad j \in D,\ k \in CS$$

3. Optimal Solution Algorithm: SMILE

The above model belongs to the class of large scale mixed integer programming formulations. At the current we solve this model by generating the corresponding MPS file from the project data base and submitting this MPS file to MINTO (Savelsbergh & Nemhauser). MINTO is a mixed integer programming solver developed at the Georgia Institute of Technology which uses in turn CPLEX or OSL as linear programming solvers. The solution from MINTO is then inserted in project data base and then displayed by CIMPEL.

3.1. General Framework

The solution method presented here works in two phases. The first phase includes preprocessing and reformulation. The second phase includes three major components: branching rules, primal heuristics, and valid inequalities. This second branch and cut phase will be done by customizing MINTO application functions.

3.1.1. Preprocessing

Preprocessing includes simplifying rows and columns by identifying redundant rows, fixing variables, increasing lower and decreasing upper bounds on variables. Many detailed preprocessing techniques described in Savelsbergh (1992) are already implemented inside MINTO. The preprocessing approach presented here focuses on simplifying rows and columns, increasing lower and decreasing upper bounds on variables from the logical implications of the problem data. Following is a list of cases where the formulation may be simplified.

- The total product flow through an arc or a node can not exceed total customer demands in the system. If the flow capacity of a warehouse type exceeds the total customer demands, the flow capacity of that warehouse type can be replaced by the total customer demands. If the production capacity of a plant exceeds the largest possible production resource units used, the plant production capacity can be replaced by the largest possible production resource units used.

- Constraint (MD) requires that each customer be within a specified maximum travel distance from the warehouse that serves it. For those warehouses located farther than the specified travel distance from a customer, the transportation channels and the product flows between those warehouses and that customer can be eliminated from the formulation without affecting the solution.

3.1.2. Reformulation

A real world problem may not require all the features of the model. Many cost components and constraints may be optional depending on the problem instance. For example, if a customer does not require a single source, the single sourcing constraints (SS1 and SS2) for that customer will not appear in the formulation. There are several cases for which part of the formulation can be changed to achieve better solution performances.

- For an existing warehouse j, constraint (DT') requires that exactly one type of warehouse be opened.

$$\sum_{l \in L} \mathrm{y}_{jl} = 1 \qquad j \in D \qquad \text{(DT')}$$

- If a transportation channel ijm doesn't have minimum number of carrier restriction and the destination customer of that channel does not require single sourcing, the (XC), (IC), (TD), and (FD) constraints can be reformulated to eliminate the channel binary variable zijm. The new formulations are then

$$\mathrm{w}_{ijm} \le \mathrm{MaxC}_{ijm} \qquad i \in B,\ j \in C,\ m \in M \qquad \text{(XC')}$$

$$\mathrm{w}_{ijm} \le \mathrm{MaxC}_{ijm} \sum_{l \in L} \mathrm{y}_{jl} \qquad i \in B \cup D,\ j \in D,\ m \in M \qquad \text{(TD')}$$

$$\mathrm{w}_{jkm} \le \mathrm{MaxC}_{jkm} \sum_{l \in L} \mathrm{y}_{jl} \qquad j \in D,\ k \in C \cup D,\ m \in M \qquad \text{(FD')}$$

$$\mathrm{w}_{ijm} \le \mathrm{MaxC}_{ijm} \mathrm{b}_i \qquad i \in B,\ j \in C \cup D,\ m \in M \qquad \text{(FP)}$$

- If the value of a particular commodity equals zero then all terms related to the inventory holding process for that commodity can be eliminated from the model.

3.1.3. Branching Rules

The proposed branching rules are listed as follows:

- First, branch on the plant binary variables bi. Within different plant binary variables, branch on the one with value closest to 0.5 first.

- Second, branch on the warehouse type binary variables yjl. Since for each warehouse site the warehouse type variables are special order sets, set all the warehouse variables with flow capacity larger than the warehouse internal flow to zero on a branch and set them to one on the other branch, i.e., one branch will be to build a warehouse of size larger than the current product flow, while the other branch will be to build a warehouse of size smaller than the current product flow. Within different warehouse sites, branch on the warehouse with the largest product flow.

- Third, branch on the single sourcing variables qjl. Since for each customer the single sourcing variables are also special order sets, follow the procedures similar to the warehouse type, but branch by transportation costs. Within different customers, branch on the customer with the largest demand.

- Forth, branch on the channel binary variables zjijm. within different channel binary variables, branch on the one with value closest to 0.5.

3.1.4. Primal Heuristics

The proposed primal heuristics for finding an upper bound of the optimal solutions is a linear programming based heuristics. Solve the LP relaxation problem; follow the order of plant, warehouse, customer, and channels in the branching rule section; fix one binary variable with value closest to one, except for the warehouse type variables fix the variable with capacity closest to the internal flow; then resolve the problem with LP. Iterate this procedure until there are no more binary variables with fractional values.

3.1.5. Valid Inequalities

Following is a set of valid inequalities that can be added to the constraints to tighten the formulation:

$$\sum_{m \in M} x_{jkmp} \leq \sum_{l \in L} DM_{kp}\, y_{jl} \qquad j \in D,\ k \in C,\ p \in P \qquad \text{(VI1)}$$

$$\sum_{m \in M} x_{ijmp} \leq DM_{jp}\, b_i \qquad i \in B,\ j \in C,\ p \in P \qquad \text{(VI1)}$$

$$\sum_{m \in M} x_{jkmp} = DM_{kp}\, q_{jk} \qquad j \in B \cup D,\ k \in CS,\ p \in P \qquad \text{(VI1)}$$

Several case studies have been completed or are in progress using the SMILE algorithm to design a variety of logistics systems. Preliminary computational results are given in the section on case studies.

4. Logistics Design Environment: CIMPEL

CIMPEL was developed to run under the Microsoft Windows operating environment, which provides the graphical user interface. A typical display screen for the design of a distribution system of a company in the continental United States is shown in the following Figure. Multiple windows allow the user to focus on different aspects of the same problem, while still maintaining the overall view and numerical statistics.

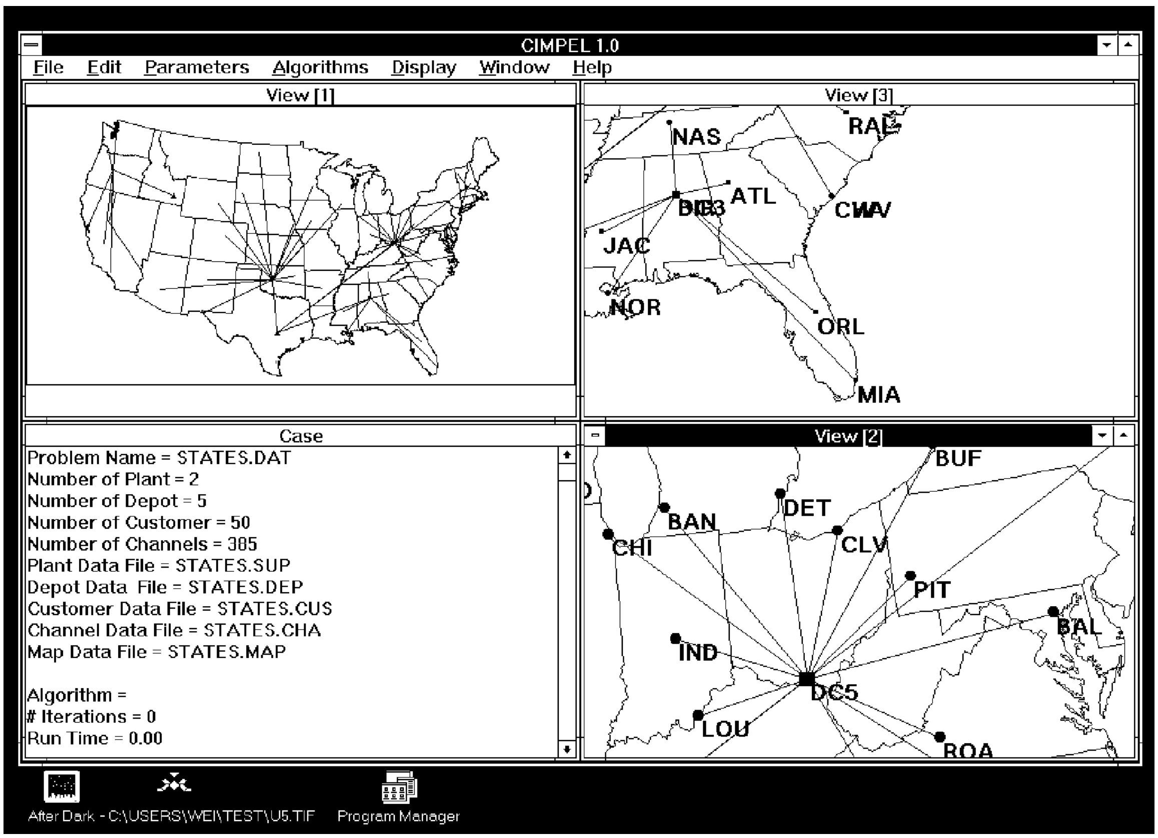

Figure 4. CIMPEL Screen Illustration

The CIMPEL environment encompasses an algorithms toolkit which includes heursitics for rapid prototyping, optimal algorithms such as SMILE for detailed design, evaluation algorithms for sensitivity analysis, and simulation for verification. All algorithms use the same underlying data base, which corresponds to the integrated logistics model, so that their results are consistent and comparable.

5. Case Studies

At the current time several case studies are being conducted with the member companies of the Material Handling Research Center, but two studies have already been concluded. The first study involves the design of the distribution network for a national hospital supply company. The second case study involves the design of the system to recycle residential plastics in the state of New Jersey. The best case recycling system was 25 % less expensive than the current recycling system. Observe that the recycling study used the same logistics model as the distribution model without any modifications, except setting the inventory values to zero. Individual runs took less than 180 minutes on an IBM RISC 6000 workstation. We are currently in the process of reducing the required computation times, while at the same time retain the general model structure. The generated problem sizes, solution algorithm properties, and computation times are given in the next two tables

Table 1. Model Sizes and Computation Times for the Case Studies

Case	# Products	# Depots	# Cust.	# Plants	# Channels
New Jersey	1	64	21	1	1408
Ohio	4	3	8	5	45
Hospital Supplies	46	7	49	46	1029
Electronics	30	25	55	30	2125
Case	# Const.	# Vars.	# Binary	# Integer	CPU Time
New Jersey	2902	1472	64	0	38 sec
Ohio	209	270	9	45	30 min
Hospital Supplies	27441	15128	1122	0	179 min
Electronics	2919	68400	150	0	99 min

Table 2. MIP Solution Properties and Computation Times (Electronics Case Study)

Software	CPU	Nodes	Depth of	LPs	Cuts
	seconds	Evaluated	B&B Tree	Solved	Generated
Minto (i)	27428	309	114	334	412
Minto (i,f at root)	21482	229	111	257	379(i),1755 (f)
Minto (B&B only)	7103	399	114	399	
Minto (g)	5913	259	44	259	
CPLEX (def. MIP)	9630	663		663	
i: Implication Inequalities					
f: Generalize Flow Covers					
g: Primal Heuristics fixing vars.					

6. Conclusions

This research has shown that the development and solution of comprehensive models for the design of integrated logistics systems is possible. Deterministic models can be solved with current state of the art mixed integer programming techniques and solvers. The solution times on powerful workstations are significant but not unacceptable. An interactive graphical front end is required to communicate the results of this model and to allow an easy sensitivity analysis. Similarly, an consistent underlying data base is required to allow for the execution of different tools and algorithms. The availability of relatively inexpensive but powerful computer workstations graphical user interfaces encourages further developments in these three directions.

References

Aikens, C.H., "Facility location models for distribution planning," European Journal of Operations Research, 22, 263-279 (1985).

Ballou, R.H. Business Logistics Management, Prentice-Hall, Inc., Englewood Cliffs (1985).

Bhaskaran, S., and Turnquist, M.A., "Multiobjective transportation considerations in multiple facility location," Transportation Research- A, 24A, 2, 139-148 (1990).

Bilde, O., and Krarup, J., "Sharp lower bounds for the simple location problem," Annals of Discrete Mathematics, 1, 79-97 (1977).

Copacino, W., and Rosenfield, D.B., "Analytic tools for strategic planning," International Journal of Physical Distribution and Materials Management, 15, 3, 47-61 (1985).

Council of Logistics Management, "What it's all about," (1990).

Daganzo, C.F., Logistics Systems Analysis, Springer-Verlag, Berlin (1991).

Delaney, R.V., "Trends in logistics and U.S. world competitiveness," Transportation Quarterly, 45, 1, 19-41 (January 1991).

Erlenkotter, D., "A dual-based procedure for uncapacitated facility location," Operations Research, 26, 992-1009 (1978).

Gao, L., and Robinson Jr., E.P., "A dual based optimization procedure for the two-echelon uncapacitated facility location problem," Naval Research Logistics, 39, 191-212 (1992).

Geoffrion, A.M., and Graves, G.W., "Multicommodity distribution system design by Benders decomposition," Management Science 20, 5, 822-844 (1971).

Hall, N.G., "A multi-item EOQ model with inventory cycle balancing," Naval Research Logistics 35, 319-325 (1988).

Ho, P., "Warehouse location with service sensitive demand," Ph.D. Dissertation (J. Perl, chairman), University of Maryland (1989).

House, R.G., and Karrenbauer, J.J., "Logistics system modeling," International Journal of Physical Distribution and Materials Management, 12, 3, 119-129 (1982).

Kearney, A.T., Measuring and Improving Productivity in Physical Distribution, National Council of Physical Distribution Management, Oak Brook, Illinois (1984).

Klincewicz, J.G., and Luss, H., "A dual-based algorithm for multiproduct uncapacitated facility location," Transportation Science, 21, 3, 198-206 (1987).

Kuehn, A.A., and Hamburger, M.J., "A heuristic program for locating warehouses", Management Science, 9, 643-666 (1963)

Magee, J.F., Copacino, W.C., and Rosenfield, D.B., Modern Logistics Management, John Wiley & Sons, new York (1985).

Moiseenko, G.E., "A multicommodity network flow problem with commodity conversion and dependent flows," Translated from Avtomatika i Telemekhanika, 11, 93-100 (1984).

Novich, N.S., "Leading edge distribution strategies," The Journal of Business Strategy, 48-53 (November/December 1990).

Perl, J., and Sirisoponsilp, S., "Distribution networks: facility location, transportation, and inventory," International Journal of Physical Distribution and Materials Management, 18, 6, 18-26 (1988).

Savelsbergh, M.W.P., "Preprocessing and probing for mixed-integer programming problem," COSOR-92-06, Eindhoven University of Technology (1992).

Simchi-Levi, D., "Hierarchical planning for probabilistic distribution systems in Euclidean spaces," Management Science, 38, 2, 198-211 (1992).

Silver, E.A., and Peterson, R., Decision Systems for Inventory Management and Production Planning, Second Edition, John Wiley & Sons, New York (1985).

Sirisoponsilp, S., "Warehouse location under multiple transportation options," Ph.D. dissertation (J. Perl, chairman), University of Maryland (1989).

Tcha, D., and Lee, B. "A branch-and-bound algorithm for the multilevel uncapacitated facility location problem," European Journal of Operational Research, 18, 35-43 (1984).

Van Roy, T.J., and Erlenkotter, D., "A dual-based procedure for dynamic facility location," Management Science, 28, 1091-1105 (1982).