A Comprehensive Model for the Concurrent Determination of Aisles and Load Stations for Aisle-Based Material Handling Systems

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ABSTRACT

We have developed a set of procedures that convert a block layout into a material handling layout by simultaneously determining the aisles and the locations of the load/unload stations for carrier based material handling systems. The procedures can be applied for several vehicle dispatching policies and for unidirectional and bi-directional aisles. We will report on computational results for standard mixed integer and linear programming solutions as well briefly discuss our heuristic solution procedures and software implementation.

Determining the set of aisles and the locations of the stations on the aisles for an aisle-based unit load material handling system is an important problem faced in facilities design. Existing research has addressed either the problem of 1) determining the aisle selection set when the station location is given, 2) the problem of determining the station location set when the aisle set is provided, 3) while accounting for the transportation costs only and ignoring fixed aisles costs, or 4) while ignoring the unloaded vehicle travel. This research develops a comprehensive network flow model for the combined problem of determining the aisle set and the station locations and accounts for the fixed as well as the transportation costs. Models for determining unloaded vehicle travel by "simulated first-come-first-served" and "minimum empty travel" dispatching policies are developed.

Preliminary studies have indicated that solving the models by standard optimization mixed integer optimization techniques for medium to large data sets requires prohibitive computer time. Initial results of some of our heurstic solution methods are presented. Results from this research are especially well suited for a "green field" design approach. The material flow network along with the block layout can then be used to perform the detailed layout of the facility.

1. INTRODUCTION

Material handling system (MHS) design is an important component of the facilities design. MHS design includes designing the material handling equipment and the material handling network. The material handling network describes the scheme of interaction between the material handling equipment and other functional components in the system. For an aisle based vehicular system, the material handling network includes the set of aisles used by the vehicles and the station locations where material gets exchanged between the MHS and the departments, stores and buffers.

This research addresses the problem of determining the material flow network for an aisle based MHS. Examples of this technology includes automated guided vehicle systems (AGVS), forklifts, industrial trucks, tow-line conveyors and people. The block layout combined with the material flow network can then be used to perform a detailed layout (Montreuil and Venkatadri, 1988) (Montreuil, 1990).

1.1. Problem Definition

The design of material handling network for an aisle based, vehicular material handling system can be stated as follows:

Given block layout, and the interdepartmental vehicular flows, determine the aisle configuration, the paths used for the movement of these flows and the location of the pickup/deposit/combined (P/D/C) stations. A station is where the material transfer occurs between the departments and the vehicles and three type of stations are identified. A pickup station for a department *i* is one from where the vehicles pick up material to be delivered from this department to other departments. At a deposit station, the vehicles drop the material that needs to enter the department. A combined station acts as both, a pickup and a deposit point. The important design parameters of the problem include the station costs, the aisle costs, and the transportation costs.

This research thus addresses the combined problem of

- determining the location and direction of the aisles and
- determining the location of the P/D/C station
- determining the vehicle paths
- determing the unloaded vehicle dispatching in some models

to support the flows (both loaded and unloaded). Unlike previous research, this research considers both the transportation and the fixed setup costs for the design of the material flow networks for aisle-based unit load systems.

1.2. Cost Parameters

There are two major types of costs associated with the problem - fixed and variable costs. The fixed costs arise from the decision of opening of aisles and P/D/C stations and are determined from capital expense, floor space, etc. The variable costs are related to the movement of the vehicles and are a function of type (loaded or unloaded) and distance of the move.

The fixed costs for the opening of P/D/C station may include the cost of any equipment required to transfer material from (to) the vehicles to (from) the department and any costs pertaining to the storage area at the department for the material and costs for aisles. The storage area cost identifies the opportunity cost of the area used by the equipment. The combined station cost need not be a sum of the pickup and deposit station costs because a transfer equipment may be able to handle both loading and unloading of the vehicles. The aisle costs may include the setup costs to account for the area required by the aisles (opportunity costs) and costs of additional equipment required to facilitate the movement of the vehicles such as laying of the wires for a wire guided AGVS. The aisle costs may also include any maintenance costs of the aisles.

The variable costs are transportation costs for the vehicle moves, including both loaded and unloaded vehicle travel, and are a function of the moved distance. The loaded and unloaded variable transportation costs need not be the same; this accounts for the difference in power required when a vehicle carries a unit load or not.

1.3. Assumptions

The following are provided as input parameters: the list of the departments, the block layout of the departments, and the loaded vehicle moves between the departments. Some assumptions related to the input parameters must be made before developing the models.

1. Each department is atomic (each department cannot be further divided into sub-departments and flows occur between departments only).

2. Material handling and movements occur in unit loads and are handled by unit load vehicles.

3. Two types of vehicle moves occur, namely loaded and unloaded vehicle moves.

4. Loaded flows represent the moves of loaded vehicles and are considered to be deterministic, i.e., there is no variation of flow with time. The models developed here do not determine the amount of loaded flows entering or leaving a particular department and have to be determined externally before the model can be applied. Some models determine the unloaded vehicle flows, while other require the unloaded vehicle flows as an input parameter. Unloaded flow times are also deterministic.

5. The material handling system is a ground-based system. All inter-departmental flows occur through the aisle set.

6. Each department is a polygon. Non-polygonal departments can be approximated by a high degree polygon.

7. The block layout for all the departments is given with the departmental areas inflated to take into account the need for aisles. If departments are required to be adjacent for other considerations (such as common tools, power requirements, etc.) then the block layout of a "super" department (one with aggregate areas and flows) should be done. This "super" department should then be split into the individual departments before using the proposed models.

8. Candidates for the aisle set lie on the boundaries of the departments.

9. P/D stations lie on the aisle set, i.e., on the boundaries of the departments.

2. LITERATURE REVIEW

2.1. Introduction

This section reviews the literature about the design of material handling networks for the aisle based, vehicular systems. Much of the available research pertains to the AGV systems. In the earliest considerations of the AGV system design, Maxwell and Muckstadt (1982) present a design methodology to determine the vehicle requirements and the flow paths for a given aisle set. Their model minimizes the number of empty vehicle trips and the total vehicle travel. Although this research does not focus on the design of the material handling networks but on vehicle routing, it sparked an interest in this problem. The literature review is presented in four parts - "Aisle set models," "Station location models" and "Material handling network models", and finally "Fixed Charge Network Flow Algorithms."

2.2. Aisle Set Models

A number of papers consider the problem of identifying the set of aisles required to support the required flows. In this problem, the block layout and the loaded from-to flows as well as the location of the stations at the departments are given. Gaskins and Tanchoco (1987) uses the transportation costs to determine the aisle set for the unidirectional flow problem. Their path based model only considers loaded flows and has quadratic terms in the objective function. Linearizing the quadratic terms requires additional constraints that increases the problem size rapidly. The solution of the model gives the direction of the aisles and the path used by each positive station-to-station flow. Their model does not consider the empty vehicle travel. Kaspi and Tanchoco (1990) modify the Gaskins and Tanchoco model to include the empty vehicle travel in the model. This path based model includes aisle feasibility constraints, unidirectionality constraints, constraints to force at least one output and one input arc for each intersection and station node, and constraints to require at least one input arc into a deposit station and at least one output arc out from a pickup station. Their model uses a branch-and-bound methodology to solve the integer program formulation to minimize vehicle travel. Sinriech and Tanchoco (1991) argue that the number of branches that need to be investigated by the solution procedure suggested by Kaspi and Tanchoco can be reduced by using the information of directional dependence between incoming and outgoing arcs at a node. Their model is similar to the Kaspi and Tanchoco model.

Gaskins et al. (1989) develop a model for determining flow paths for free-ranging automated guided vehicles. Station locations are given for this multicommodity, arc based network model. This model contains the network conservation equations and constraints to limit the number of lanes allowed in an aisle. This model determines the flow paths for the flows and the number of lanes that need to be opened in an aisle. The number of lanes depends on the flow in an aisle and the maximum number of trips allowed per lane.

Chhajed et al. (1992) address the problem of determining a minimum length rectilinear network. The P/D station locations are given for this arc based model. The objective function has a fixed charge costs on the aisles but does not include the transportation costs. The constraints for this model include the network conservation equations and the aisle feasibility equations. A Lagrangian relaxation of the rectilinear network problem is used to determine shortest path problems. They introduce a heuristic based on these shortest path problems and determine the gap between the heuristic solution and the optimal solution. They also present an improvement heuristic to generate a better solution.

Kouvelis et al. (1992) present five heuristics for the design of unidirectional flow paths for AGVS. Here too, only the transportation costs are considered. The various heuristics are based on the shortest paths and the only difference being in the way the flows are sorted to fix the directions of the flow segments. The flow assignments are base on decreasing order of from-to flows, decreasing order of production volumes of various products, decreasing order of the scalar product of production volume and shortest distance, and, decreasing order of the scalar product of production volume with shortest distance with recomputing the distance after each assignment. The last heuristic routes first the flow that imposes the least limitations on the routing of the remaining flows. They also implement two simulated annealing heuristics for the flow path design problem.

2.3. Station Location Models

A set of literature looks only at the problem of the station locations. Montreuil and Ratliff (1988) address this problem when only the block layout and the from-to flows are given. Their optimization model minimizes the sum of the rectilinear distance traveled by all inter-station flows, and the model allows multiple stations per department. They solve the P/D station location problem for each coordinate separately by an iterative algorithm. At each iteration of the algorithm, a node is created for each department that can have a station located at that coordinate. This is determined from the extent of the department in the block layout. Two additional nodes are then created - the first node represents all nodes that need to be strictly to the left of the current coordinate, and the second node that represents all the nodes that need to be strictly to the right of the current coordinate. A minimum cut is then determined for this network to decide the candidate stations that need to be located at the current coordinate. The process continues till all the stations are located. This method may sometimes suggest locating the P/D station at a place other than the perimeter of the department.

Kim et al. (1993) look at the problem of locating the P/D stations when the aisles are assumed to be bi-directional and the departments are rectangular. The problem is modeled as a facility location problem with the objective to minimize the travel distance in the MHS. They look at two general cases of the problem. In the first case, the candidate P/D station locations are discrete points at the perimeter of the department and given. In the second case, the station locations are allowed to lie anywhere on the perimeter of a department. They develop O(mn) heuristics for both cases.

Kiran and Tansel (1989) address the problem of location of the P/D stations, given the aisle set and the direction of the aisles in it. Their model considers only the transportation cost and does not distinguish between pickup and deposit stations. Instead, there is only one kind of station. If stations are allowed to be located at the intersection nodes then the authors present an

O(n) algorithm to determine the optimal station locations where n is the number of intersection nodes. If however, the stations cannot be located at the intersection nodes, then they present an O(kn) algorithm, where k is the number of intersection nodes whose indegree or outdegree or both are greater than one.

2.4. Material Handling Network Models

A set of literature deal with the combined problem of determining both the aisle set and the locations of the P/D stations. Goetz and Egbelu (1990) present a model to minimize the total distance traveled. The objective function of this model includes transportation costs and has quadratic terms. Additional constraints are required to linearize these quadratic terms. Other constraints in the model are the ones from the Gaskins and Tanchoco (1987) model. This model considers unidirectional aisles and major flows (flows that are significant in terms of magnitude). Use of the major flows reduces the number of constraints and so larger problems can be solved. Candidate locations for the P/D stations are assumed to be given for the model. The solution determines the best location for the stations and the unidirectional aisle set for the major flows. They use MPSX - MIP to solve a three department case, but the computation times are not reported.

Tanchoco and Sinriech (1992) address the problem of determining an optimal single loop for the movement of the vehicles. The objective is to minimize the travel distance. The solution is obtained in two stages. In the first stage a "best" loop is obtained by enumerating all the possible loops and eliminating inferior loops by dominance conditions. Once the "best" loop is obtained, they solve an integer programming model to determine the best location of the P/D stations. They claim that this two stage process yields optimal solutions to the transportation problem.

2.5. Fixed Charge Network Algorithms

Khang and Fujiwara (1991) address the problem of capacitated fixed charge minimum cost networks. The objective function has a fixed charge component and a variable charge component. This arc based model is for a single commodity and the network has a single supply node with all other nodes having positive demands. The constraints include network flow balance and capacity bounds on the arcs. Lagrange relaxation of the model is shown to form a minimum spanning tree to yield a lower bound to the problem. They also present a scaling heuristic to find a feasible and approximate solution that is an upper bound to the problem.

Palekar et al. (1990) introduce a path based fixed cost transportation model The objective function of this model has variable component as well as the fixed charge component. The model has constraints to force flow out of the source nodes, constraints to force flow into the destination nodes, and arc existence constraints linked to the flow on the arc. They introduce a branch and bound scheme to solve the problem. In this method, a lower bound on the additional cost incurred to obtain an integer solution from the current LP solution is calculated. This lower bound is used for selecting the branching rules.

Crainic et al. (1993) present a branch and bound scheme for a multicommodity location-allocation problem. This problem has additional balancing constraints that require that the empty containers move back to the depot. Bounds are computed by using a reformulation of the weak relaxation of the problem as a multicommodity flow problem. They also investigate various branching rules and report that hybrid rules perform better than simple ones.

Van Roy and Wolsey (1985) describe a family of valid inequalities for the fixed charge network problems. From computational results they conclude that automatic cut generation combined with branch and bound can solve a variety of reasonably sized MIP problems. Leung and Magnanti (1989) introduce a family of facets for a single commodity capacitated facility location problem with constant capacity for all the plants. They identify facets and valid inequality for the single sourcing version of the problem by applying results of vertex packing problem.

3. MODEL FORMULATION

3.1. Introduction

The following section defines some of the terms used in the development of the model. The methods to handle the unloaded flows in the model are then presented. Lastly, notations are defined and the model developed.

3.2. Network Geometry

A department boundary is represented by segments which in turn are defined by the corner nodes and the intersection nodes. The set of corner nodes are the nodes essential to define the geometry of the department. An intersection node for a department *d* is one that is the corner node of an adjacent department but is not necessary to define the geometry of department *d*. For example, in Figure 1, nodes 1, 2, 4, and 5 are the corner nodes for department M while node 3 is its intersection node.

3.3. Reducing Solution Space

The P/D/C stations are allowed to lie along the boundary of a department; thus, the solution space for the location of the stations are the segments that define the department's boundaries. This indicates that the solution for the station location problem is in a continuous space. However, this space can be reduced to a discrete number of points by the following theorem.

Theorem 1:

If the aisles are only allowed to be unidirectional, in the optimal solution, the P/D station will be located at the end nodes for the department's aisle segments.

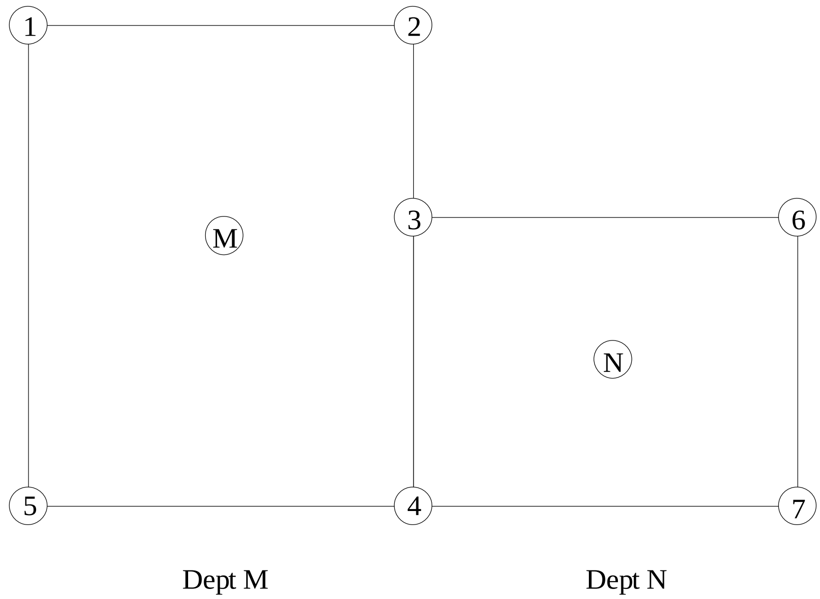

Figure 1 - An example block layout

3.4. Unloaded Flows

The loaded flows determine the vehicle requirements at the origin nodes of the flow paths and the purpose of unloaded flows is to make the empty vehicles available at the right place in the right quantity to maintain the conservation of vehicles. The source for the unloaded flows are the destination nodes for loaded vehicles and the destination for the unloaded flows are the pickup nodes for the loaded flows. The sum of all the incoming unloaded flows must equal the sum of all the leaving loaded flows at the pickup and combined station locations. The sum of all the incoming loaded flows must equal the sum of all the outgoing flows at the deposit and the combined stations. The unloaded flows are determined by two methods - minimum cost location strategy, and factoring strategy. The latter strategy models the long run first in first out (FIFO) method of dispatching the empty vehicles and is more realistic than the first strategy where, the empty vehicles are always dispatched to the nearest location

3.4.1. Minimum Cost

In the minimum cost location (MC) strategy, the unloaded flow quantities are determined within the model. The dispatching of the empty vehicles is dependent on the set of opened aisles. In this strategy, all the unloaded flows are considered to be a single commodity. The unloaded flow network has multiple sources (corresponding to the locations where the loaded flows end) and multiple destinations (corresponding to the locations where the loaded flows originate). Solving this model involves in assigning the flows to the destination nodes that is "least expensive" to the source locations. Here, "cost" reflects the fixed setup costs for opening any additional aisle segments solely for the unloaded travel as well as the transportation costs.

3.4.2. Factoring

In the factoring strategy, the unloaded flow quantities need to be determined outside the model. In this method, a fixed percentage of all the outgoing (empty vehicle) flows is sent from a particular deposit station to the various pickup stations. These percentages are determined by the empty vehicle requirements of each department. Table 1 gives an example of loaded flows and Table 2 is its corresponding unloaded flows by method of factoring.

Table 1 - An example of loaded flows

To Department / From Department	A	B	C	Total Outflow
A		5		5
B	3		7	10
C	10			10
Total Inflow	13	5	7	25

Table 2 - Unloaded flows by factoring for flows in Table 1

To Department / From Department	A	B	C	Total Outflow
A	3	5	5	13
B	1	2	2	5
C	1	3	3	7

The fractional values are rounded to the nearest integer in sequence of smallest rounding adjustment so that the total rounding adjustment equals zero. For the first row in Table 2,

$f_{AA} = 13 \times \frac{5}{25} = 2.6$ which on nearest integer rounding yields 3 vehicles with a remainder of $(-0.4.)$

$f_{AB} = 13 \times \frac{10}{25} = 5.2.$ $5.2 + (-0.4)$ equals 4.8, which on nearest integer rounding yields 5 vehicles with a remainder of $(-0.2.)$

$f_{AC} = 13 \times \frac{10}{25} = 5.2.$ $5.2 + (-0.2)$ yields exactly 5 vehicles for the last destination.

The method of factoring approximates the long run FIFO (first in first out) dispatching policy. A more sophisticated approximation uses a mixed integer programming formulation to round the fractional values to integer vehicle numbers.

3.5. Arc Capacities

Arc capacities are introduced in the model to limit the vehicular traffic occuring through an aisle. These arc capacities are a function of the vehicle and system operating characteristics. The capacity on an arc (ij) is given by

$$\mathrm{CAP}_{\mathrm{ij}} = \frac{\mathrm{PH} * (1 - \mathrm{CF})}{\frac{l_{ij}}{S_v} + t_{\min}} \tag{1}$$

where

PH is the planning horizon,

CF is congestion factor that takes a value between 0 and 1,

l_{ij} is the length of arc (ij)

S_v is the speed of the vehicle, and

$t_{\min}$ is the minimum time required for a vehicle to enter an aisle.

3.6. Variables and Definitions

A simple block layout of departments M and N is given in Figure 1. The intersection and corner points of the departments are nodes 1 through 7. The block layout can be transformed to a directed graph, with nodes corresponding to potential P/D/C nodes and the intersection nodes. The directed arcs are the potential aisles. The directed graph resulting from the two department block layout of Figure 1 is given in Figure 2 which will be used to explain the variables and sets for the model.

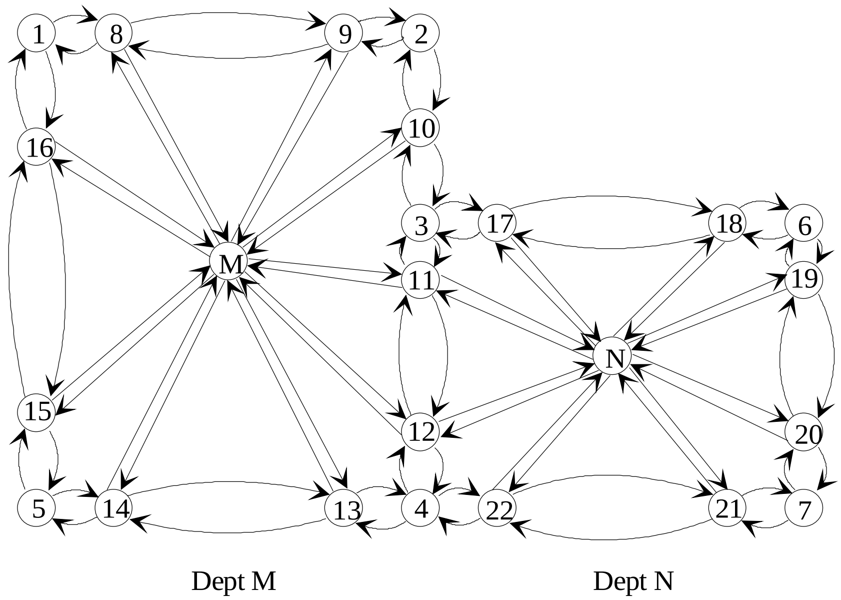

Figure 2 - Node and edge representations of the departments in Figure 1

The nodes numbered 8 through 22 are the candidate P/D/C station locations for the two departments which are located a δ distance away from the corner and intersection nodes. Nodes M and N represents the department M and N respectively. All arcs other than the ones incident to or leaving the nodes M and N are the candidates for aisles. The list of variables and definitions for the model development follows:

F_{lab} - loaded flow from department *a* to department *b*.

F_{uab} - unloaded flow from department *a* to department *b*. This is a parameter in the factoring strategy and is determined in the minimum cost strategy.

N - total number of departments.

S - $\{a : F_{lab} > 0,\ b = 1, \cdots, N\}$. i.e., source nodes for loaded flows.

T - $\{b : F_{lab} > 0,\ a = 1, \cdots, N\}$. i.e., sink nodes for the loaded flows

δ - the minimum distance from a corner point of a department within which a station (deposit, pickup, or combined) cannot be located.

N' - the set of nodes. A node is defined to be a corner point of a department and possible locations where (P/D/C) stations may be located. In Figure 2, N' = {1,…, 22}

N'' - nodes that need to be added for departments which have flow between them. The set $N'' = S \cup T$. In Figure 2, if flows enter or leave departments m and n, then $N'' = \{M, N\}$.

E - the set of arcs defined on the set of nodes N'. All the arcs with arrow heads except the ones that originate or enter the nodes M and N in Figure 2 form the members of this set.

E'' - arc that need to be added from the node representing the department to the nodes that represent the P/D/C stations. In Figure 2, the members of this set are the arcs originating from, and entering into the nodes M and N. We label the arcs leaving the nodes M and N as *leaving arcs* and the arcs entering these nodes as the *entering arcs* in the rest of this paper. The nodes that define these arcs are labeled *leaving nodes* and *entering nodes* respectively.

$\mathcal{N}$ - $N' \cup N''$

$\mathcal{E}$ - $E \cup E'$

(ij) - the arc from node *i* to node *j*.

l_{ij} - length of arc (ij)

np_d - maximum number of pickup stations allowed for department *d*.

nd_d - maximum number of deposit stations allowed for department *d*.

z_{ij} - 1, if arc (ij) is open; 0, otherwise.

n_{ij} - parameter used to control directionality of arc (ij). n_{ij} takes the value 1 if only unidirectional aisles are allowed, and 0 if no aisle are allowed on arc (ij).

x_{labij} - loaded flow from department *a* to department *b* on arc (ij).

x_{uij} - unloaded flow on arc (ij). These variables are used in the MC strategy for the unloaded flows.

x_{uabij} - unloaded flow from department *a* to department *b* on arc (ij). These variables are used in the factoring strategy for the unloaded flows.

c - one-time cost per unit length of building an aisle.

t_l - transportation cost per unit distance traveled per unit of loaded flow.

t_u - transportation cost per unit distance traveled per unit of loaded flow.

py_{di} - 1, if a pickup station for department *d* is opened at node *i*; 0, otherwise.

dy_{di} - 1, if a deposit station for department *d* is opened at node *i*; 0, otherwise.

cy_{di} - 1, if a combined station for department *d* is opened at node *i*; 0, otherwise.

c_p - cost of locating a pickup station.

c_d - cost of locating a deposit station.

c_c - cost of locating a combined pickup/deposit station. $c_c \geq c_p$, and $c_c \geq c_d$.

P_d - set of nodes that are potential locations for pickup stations of department *d*. In Figure 2, for department m, these are {8,…, 16}.

D_d - set of nodes that are potential locations for deposit stations of department *d*. In Figure 2, for department n, these are {11, 12, 17,…, 22}.

C_d - set of nodes that are potential locations for combined stations of department *d*. In Figure 2, for department m, these are { 8,…, 16}.

D, A, B - nodes that represent departments *d*, *a*, *b*.

3.7. Material Handling Network Models

This section introduces the mathematical formulation of the problem. Two formulations are developed corresponding to the minimum cost model and the factoring model. Both formulations have the structure of a multicommodity, fixed-charge network problem with side constraints that require flow balance between commodities. These side constraints ensure that loaded vehicles entering a station equals the leaving unloaded vehicles at the station nodes.

3.7.1. Minimum Cost Model

The model for the minimum cost (MC) dispatching policy that has the in-built despatching scheme for the unloaded flows is presented in this section. Both primal and dual formulations of the model follow.

Primal

Minimize:

$$\theta_{IP}^{n} = \sum_{a\in S}\sum_{i\in P_a} c_p\, py_{ai} + \sum_{b\in T}\sum_{i\in D_b} c_d\, dy_{bi} + \sum_{d\in S\cup T}\sum_{i\in C_d} c_c\, cy_{di} + \sum_{(ij)\in E} c_l l_{ij} z_{ij} + \sum_{a\in S}\sum_{b\in T}\sum_{(ij)\in E} t_l l_{ij} x_{labij} + \sum_{(ij)\in E} t_u l_{ij} x_{uij} \qquad (2)$$

Subject to:

<u>Constraints for Loaded Flows:</u>

$$-\mathrm{L}_a: \quad \sum_{i\in P_a} \mathrm{py}_{ai} + \sum_{i\in C_a} \mathrm{cy}_{ai} \le \mathrm{np}_a \qquad \forall a \in S \qquad (3)$$

$$-\mathrm{E}_b: \quad \sum_{i\in D_b} \mathrm{dy}_{bi} + \sum_{i\in C_b} \mathrm{cy}_{bi} \le \mathrm{nd}_b \qquad \forall b \in T \qquad (4)$$

$$-\mathrm{G}_{abij}: \quad \mathrm{x}_{labij} - \mathrm{F}_{lab}\mathrm{z}_{ij} \le 0 \qquad \forall (ij) \in E,\ a \in S,\ b \in T \qquad (5)$$

$$\mathrm{H}_{ab}: \quad \sum_{j\in P_a \cup C_a} \mathrm{x}_{labAj} = \mathrm{F}_{lab} \qquad \forall a \in S,\ b \in T \qquad (6)$$

$$\mathrm{K}_{ab}: \quad \sum_{j\in D_b \cup C_b} \mathrm{x}_{labjB} = \mathrm{F}_{lab} \qquad \forall a \in S,\ b \in T \qquad (7)$$

$$\mathrm{M}_{abk}: \quad \sum_{(ik)\in \mathcal{E}} \mathrm{x}_{labik} - \sum_{(kj)\in \mathcal{E}} \mathrm{x}_{labkj} = 0 \qquad \forall k \in N',\ a,b \ni F_{lab} > 0 \qquad (8)$$

$$-\mathrm{N}_{abAj}: \quad \mathrm{x}_{labAj} - \mathrm{F}_{lab}\mathrm{py}_{aj} - \mathrm{F}_{lab}\mathrm{cy}_{aj} \le 0 \qquad \forall a \in S,\ b \in T,\ j \in P_a \cup C_a \qquad (9)$$

$$-\mathrm{Q}_{abBi}: \quad \mathrm{x}_{labiB} - \mathrm{F}_{lab}\mathrm{dy}_{bi} - \mathrm{F}_{lab}\mathrm{cy}_{bi} \le 0 \qquad \forall a \in S,\ b \in T,\ i \in D_b \cup C_b \qquad (10)$$

<u>Constraints related to unloaded flows:</u>

$$-\mathrm{R}_{ij}: \quad \mathrm{x}_{uij} - \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{z}_{ij} \le 0 \qquad \forall (ij) \in E' \qquad (11)$$

$$\mathrm{S}_{Bj}: \quad \mathrm{x}_{uBj} - \sum_{a\in S} \mathrm{x}_{labjB} = 0 \qquad \forall b \in T,\ j \in D_b \cup C_b \qquad (12)$$

$$\mathrm{V}_{Aj}: \quad \mathrm{x}_{ujA} - \sum_{b\in T} \mathrm{x}_{labAj} = 0 \qquad \forall a \in S,\ j \in P_a \cup C_a \qquad (13)$$

$$\mathrm{Y}_k: \quad \sum_{(ik)\in \mathcal{E}} \mathrm{x}_{uik} - \sum_{(kj)\in \mathcal{E}} \mathrm{x}_{ukj} = 0 \qquad \forall k \in N' \qquad (14)$$

$$-\mathrm{A}_{ij}: \quad \mathrm{z}_{ij} + \mathrm{z}_{ji} \le n_{ij} \qquad (ij) \in E \qquad (15)$$

$$\mathrm{py}_{ai} \in \{0,1\} \qquad \forall a \in S,\ i \in P_a \qquad (16)$$

$$\mathrm{dy}_{bi} \in \{0,1\} \qquad \forall b \in T,\ i \in D_b \qquad (17)$$

$$\mathrm{cy}_{di} \in \{0,1\} \qquad \forall d \in S \cup T,\ i \in C_d \qquad (18)$$

$$\mathrm{z}_{ij} \in \{0,1\} \qquad \forall (ij) \in E' \qquad (19)$$

$$\mathrm{x}_{labij},\ \mathrm{x}_{uij} \in Z_+ \qquad \forall (ij) \in E',\ a,\ b \ni \mathrm{F}_{lab} > 0 \qquad (20)$$

The x variables represent vehicles and to prevent fractional vehicles in the solution, these are required to be integer variables. For the linear relaxation, the binary and general integer variables above are relaxed to

$$0 \le \mathrm{py}_{ai} \le 1 \qquad \forall\, a \in S,\ i \in P_a \tag{16a}$$

$$0 \le \mathrm{dy}_{bi} \le 1 \qquad \forall\, b \in T,\ i \in D_b \tag{17a}$$

$$0 \le \mathrm{cy}_{di} \le 1 \qquad \forall\, d \in S \cup T,\ i \in C_d \tag{18a}$$

$$0 \le \mathrm{z}_{ij} \quad \le 1 \qquad \forall\, (ij) \in E' \tag{19a}$$

$$\mathrm{x}_{labij} \ge 0,\ \mathrm{x}_{uij} \ge 0 \qquad \forall\, (ij) \in E',\ a,\ b \ni \mathrm{F}_{lab} > 0 \tag{46a}$$

The first three terms in the objective function are the setup costs for the pickup, deposit and combined stations respectively. The fourth term is the setup cost for the aisles and includes maintenance cost if any. The last two terms are the loaded and unloaded transportation costs respectively. Equation (2) constraints the number of pickup and combined stations that can be opened for a department while equation (3) does the same for the deposit and combined stations. Equation (4) requires aisles to be opened to allow loaded flow travel. Equation (5) requires the loaded flows to leave the appropriate department while equation (6) ensures that these flows reach the correct destination department. Equation (7) is the loaded flow balance at the nodes. Equation (8) indicates that a loaded flow can leave a department through its pickup and combined stations only, while equation (9) indicates that the loaded flow need to enter the department through its deposit and combined stations. Equation (10) requires aisle to be opened to allow unloaded flow travel. Equation (11) is the flow balance between the loaded and unloaded flows at the D/C stations; equation (12) the same for the P/C stations. Equation (13) is the unloaded flow balance at all nodes and equation (14) is the aisle directionality constraint.

Dual

The dual formulation of the linear relaxation of the above model is as follows:

Maximize:

$$\omega_{IP}^{n} = \quad -\sum_{a\in S} \mathrm{np}_a \mathrm{L}_a - \sum_{b\in T} \mathrm{nd}_b \mathrm{E}_b + \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{H}_{ab} + \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{K}_{ab} - \sum_{(ij)\in E'} n_p \mathrm{A}_{ij} \tag{21}$$

Subject to:

$$\mathrm{py}_{aj}:\ -\mathrm{L}_a + \sum_{b\in T} \mathrm{F}_{lab}\mathrm{N}_{abAj} \quad \le \ \mathrm{c}_p \qquad \forall\ a \in S,\, j \in P_a \tag{22}$$

$$\mathrm{dy}_{bj}:\ -\mathrm{E}_b + \sum_{a\in S} \mathrm{F}_{lab}\mathrm{Q}_{abBj} \quad \le \ \mathrm{c}_d \qquad \forall\ b \in T,\, j \in D_b \tag{23}$$

$$\mathrm{cy}_{dj}:\ -\mathrm{L}_a + \sum_{b\in T} \mathrm{F}_{lab}\mathrm{N}_{abAj} \quad \le \ \mathrm{c}_c \qquad \forall\ j \in C_a,\, a \in S \setminus T \tag{24}$$

$$-\mathrm{E}_b + \sum_{a\in S} \mathrm{F}_{lab}\mathrm{Q}_{abBj} \quad \le \quad \mathrm{c}_c \qquad \forall \ \ j\in C_b,\ b\in T\setminus S \tag{25}$$

$$-\mathrm{L}_d - E_d + \sum_{b\in T} \mathrm{F}_{ldb}\mathrm{N}_{dbDj} + \sum_{a\in S} \mathrm{F}_{lad}\mathrm{Q}_{adDj} \quad \le \quad \mathrm{c}_c \qquad \forall \ \ j\in C_d,\ d\in S\cap T \tag{26}$$

$$\mathrm{x}_{labij}:\ -\mathrm{G}_{abij} + \mathrm{M}_{abj} - \mathrm{M}_{abi} \quad \le \quad \mathrm{t}_l \mathrm{l}_{ij} \qquad \forall \ \ a\in S,\ b\in T,\ a,b \ni \mathrm{F}_{lab} > 0,\ (ij)\in E \tag{27}$$

$$\mathrm{x}_{labAj}:\ \mathrm{M}_{abj} + \mathrm{H}_{ab} - \mathrm{N}_{abAj} - \mathrm{V}_{Aj} \quad \le \quad 0 \qquad \forall \ \ a\in S,\ b\in T,\ a,b \ni \mathrm{F}_{lab} > 0,\ j\in P_a \cup C_a \tag{28}$$

$$\mathrm{x}_{labjB}:\ \mathrm{K}_{ab} - \mathrm{Q}_{abBj} - \mathrm{S}_{Bj} - \mathrm{M}_{abj} \quad \le \quad 0 \qquad \forall \ \ a\in S,\ b\in T,\ a,b \ni \mathrm{F}_{lab} > 0,\ j\in D_b \cup C_b \tag{29}$$

$$\mathrm{z}_{ij}:\ \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{R}_{ij} + \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{G}_{abij} - \mathrm{A}_{ij} \quad \le \quad \mathrm{cl}_{ij} \qquad \forall \ \ (ij)\in E \tag{30}$$

$$\mathrm{x}_{uij}:\ -\mathrm{R}_{ij} + \mathrm{Y}_j - \mathrm{Y}_i \quad \le \quad \mathrm{t}_u \mathrm{l}_{ij} \qquad \forall \ \ (ij)\in E \tag{31}$$

$$\mathrm{x}_{uAj}:\ \mathrm{Y}_j + \mathrm{S}_{Aj} \quad \le \quad 0 \qquad \forall \ \ j\in D_b \cup C_b \tag{32}$$

$$\mathrm{x}_{ujB}:\ \mathrm{V}_{Bj} - \mathrm{Y}_j \quad \le \quad 0 \qquad \forall \ \ j\in P_a \cup C_a \tag{33}$$

$$\mathrm{L}_a \ge 0,\ E_b \ge 0, \mathrm{G}_{abij} \ge 0,\ \mathrm{N}_{abAj} \ge 0,\ \mathrm{Q}_{abBj} \ge 0,\ \mathrm{R}_{ij} \ge 0,\ \mathrm{A}_{ij} \ge 0$$

$$\mathrm{H}_{ab},\ \mathrm{K}_{ab},\ \mathrm{M}_{abk},\ \mathrm{S}_{ij},\ \mathrm{V}_{ij},\ \mathrm{Y}_k \ \text{unrestricted}$$

3.7.2. Factoring Model

The model with the factoring strategy for despatching the unloaded flows is presented in this section. The from-to flows for the empty vehicle trips need to be determined outside the model and is one of it's inputs. This model differs from the minimum cost model in that each entry in the from-to flow for the unloaded flow is a commodity in the model. The primal of the model is first presented followed by the dual of the formulation.

Primal

Minimize:

$$\theta_{IP}^{f} = \sum_{a\in S}\sum_{i\in \mathrm{P}_a} \mathrm{c}_p \mathrm{py}_{ai} + \sum_{b\in T}\sum_{i\in D_b} \mathrm{c}_d \mathrm{dy}_{bi} + \sum_{d\in S\cup T}\sum_{i\in C_d} \mathrm{c}_c \mathrm{cy}_{di} + \sum_{(ij)\in E} \mathrm{c}_l \mathrm{l}_{ij}\mathrm{z}_{ij} + \sum_{a\in S}\sum_{b\in T}\sum_{(ij)\in E} \mathrm{t}_l \mathrm{l}_{ij}\mathrm{x}_{labij} + \sum_{a\in T}\sum_{b\in S}\sum_{(ij)\in E} \mathrm{t}_u \mathrm{l}_{ij}\mathrm{x}_{uabij} \tag{34}$$

Subject to:

Equations (2) - (9), (14)

Constraints for Unloaded Flows:

$-\mathrm{R}_{abij}$: $\quad x_{uabij} - F_{uab} z_{ij} \leq 0 \qquad \forall (ij) \in E, a, b \ni F_{uab} > 0 \qquad (35)$

I_{ab}: $\quad \sum_{j \in D_a \cup C_a} x_{uabAj} = F_{uab} \qquad \forall\ a, b \ni F_{uab} > 0 \qquad (36)$

J_{ab}: $\quad \sum_{j \in P_b \cup C_b} x_{uabjB} = F_{uab} \qquad \forall\ a, b \ni F_{uab} > 0 \qquad (37)$

T_{abk}: $\quad \sum_{(ik) \in \mathcal{E}} x_{uabik} - \sum_{(kj) \in \mathcal{E}} x_{uabkj} = 0 \qquad \forall\ k \in N', a, b \ni F_{uab} > 0 \qquad (38)$

F_{aAj}: $\quad \sum_{\{b: F_{uab} > 0\}} x_{uabAj} - \sum_{\{b: F_{lba} > 0\}} x_{lbajA} = 0 \qquad \forall\ a = 1, \ldots, \mathrm{N},\ j \in P_a \cup C_a \qquad (39)$

W_{aAj}: $\quad \sum_{\{b: F_{lab} > 0\}} x_{labAj} - \sum_{\{b: F_{uba} > 0\}} x_{ubajA} = 0 \qquad \forall\ a = 1, \ldots, \mathrm{N},\ j \in D_a \cup C_a \qquad (40)$

$py_{ai} \in \{0,1\} \qquad \forall\ a \in S,\ i \in P_a \qquad (41)$

$dy_{bi} \in \{0,1\} \qquad \forall\ b \in T,\ i \in D_b \qquad (42)$

$cy_{di} \in \{0,1\} \qquad \forall\ d \in S \cup T,\ i \in C_d \qquad (43)$

$z_{ij} \in \{0,1\} \qquad \forall (ij) \in E' \qquad (44)$

$x_{labij} \in Z_+ \qquad \forall (ij) \in E',\ a,\ b \ni \mathrm{F}_{lab} > 0 \qquad (45)$

$x_{uabij} \in Z_+ \qquad \forall (ij) \in E',\ a,\ b \ni \mathrm{F}_{uab} > 0 \qquad (46)$

The linear relaxation of the equations (41 - 46) are

$py_{ai} \in \{0,1\} \qquad \forall\ a \in S,\ i \in P_a \qquad (41a)$

$dy_{bi} \in \{0,1\} \qquad \forall\ b \in T,\ i \in D_b \qquad (42a)$

$cy_{di} \in \{0,1\} \qquad \forall\ d \in S \cup T,\ i \in C_d \qquad (43a)$

$z_{ij} \in \{0,1\} \qquad \forall (ij) \in E' \qquad (44a)$

$x_{labij} \in Z_+ \qquad \forall (ij) \in E',\ a,\ b \ni \mathrm{F}_{lab} > 0 \qquad (45a)$

$x_{uabij} \in Z_+ \qquad \forall (ij) \in E',\ a,\ b \ni \mathrm{F}_{uab} > 0 \qquad (46a)$

The last term in equation (33) depicts the multicommodity representation of the unloaded flows. Equation (34) requires an aisle to be opened to allow unloaded flow travel. Equation (35) requires the unloaded flows to leave the appropriate department, while equation (36) indicates the correct unloaded flow need to enter the department. Equation (37) is the unloaded flow balance at the intermediate nodes. Equation (38) requires that all the loaded vehicles leaving a pickup or combined station must equal the empty vehicles entering that station while equation (39) ensures that all the arriving loaded vehicles at the deposit and combined station leave as empty vehicles.

Dual

The dual formulation of the linear relaxation of the above model is as follows:

Maximize:

$$\omega_{IP}^{f} = -\sum_{a\in S} \mathrm{np}_a \mathrm{L}_a - \sum_{b\in T} \mathrm{nd}_b \mathrm{E}_b + \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{H}_{ab} + \sum_{a\in S}\sum_{b\in T} \mathrm{F}_{lab}\mathrm{K}_{ab} - \sum_{(ij)\in E'} n_p \mathrm{A}_{ij} + \sum_{a\in SS}\sum_{b\in ST} \mathrm{F}_{uab}\mathrm{J}_{ab} + \sum_{a\in SS}\sum_{b\in ST} \mathrm{F}_{uab}\mathrm{I}_{ab} \quad (47)$$

Subject to:

Equations (16) - (21), (24)

$$\mathrm{x}_{labAj}: \mathrm{M}_{abj} + \mathrm{H}_{ab} - \mathrm{N}_{abAj} - \mathrm{W}_{aAj} \le 0 \quad \forall\ a\in S,\ b\in T,\ a,b \ni \mathrm{F}_{lab} > 0,\ j \in P_a \cup C_a \quad (48)$$

$$\mathrm{x}_{labjB}: \mathrm{K}_{ab} - \mathrm{Q}_{abBj} - \mathrm{F}_{bjB} - \mathrm{M}_{abj} \le 0 \quad \forall\ a\in S,\ b\in T,\ a,b \ni \mathrm{F}_{lab} > 0,\ j \in D_b \cup C_b \quad (49)$$

$$\mathrm{x}_{uabij}: -\mathrm{R}_{abij} + \mathrm{T}_{abj} - \mathrm{T}_{abi} \le \mathrm{t}_u \mathrm{l}_{ij} \quad \forall\ (ij) \in E \quad (50)$$

$$\mathrm{x}_{uabjB}: \mathrm{J}_{ab} + \mathrm{W}_{bBj} - \mathrm{T}_{abj} \le 0 \quad \forall\ j \in D_b \cup C_b \quad (51)$$

$$\mathrm{x}_{uabAj}: \mathrm{I}_{ab} + \mathrm{F}_{aAj} + \mathrm{T}_{abj} \le 0 \quad \forall\ j \in P_a \cup C_a \quad (52)$$

$$\mathrm{L}_a \ge 0,\ \mathrm{E}_b \ge 0,\ \mathrm{G}_{abij} \ge 0,\ \mathrm{N}_{abAj} \ge 0,\ \mathrm{Q}_{abBj} \ge 0,\ \mathrm{A}_{ij} \ge 0,\ \mathrm{R}_{abij} \ge 0$$

$\mathrm{H}_{ab},\ \mathrm{I}_{ab},\ \mathrm{J}_{ab},\ \mathrm{K}_{ab},\ \mathrm{M}_{abk},\ \mathrm{T}_{abk},\ \mathrm{W}_{aAj},\ \mathrm{F}_{aAj}$ unrestricted

3.8. Model Features

The models introduced in section 3.7. are fixed charge (represented by the cost coefficients associated with the aisles and P/D/C station decision variables), multicommodity flows (represented by the loaded and unloaded flows) problem with additional constraints to ensure the flow balance between the loaded and the unloaded vehicles at the P/D/C stations.

The presented models determines both the aisles set and the station locations at the same time and these are more comprehensive than the ones available in literature which attempts only one of the two problems (Gaskins, 1987) (Gaskins, 1989) (Kiran, 1989) (Kaspi, 1990) (Sinriech, 1991) (Kim, 1993). The proposed models, besides including the variable transportation costs, also includes fixed setup costs for the opening of the aisles and stations. This is an improvement over existing models in literature where only the variable transportation costs are included (Gaskins, 1987) (Gaskins, 1989) (Kiran, 1989) (Kaspi, 1990) (Sinriech, 1991) (Kim, 1993) (Goetz, 1990). Further, the objective function does not have any quadratic terms in the model which is the case with some of the existing models in literature (Gaskins, 1987) (Kaspi, 1990) (Sinriech, 1991) (Goetz, 1990).

The proposed models accounts for all the loaded flows and not just "important" flows (Goetz, 1990). Further, the empty vehicles are completely accounted for in the model. Empty vehicles despatching rules such as FIFO are included in the model. The proposed models consider the shape of the departments and prevents the vehicles from traversing through the departments as opposed to the rectilinear metric model which allows the vehicles the travel through the departments (Montreuil, 1988) (Chhajed, 1992) and may be unrealistic.

4. SOLUTION APPROACH

This section covers our initial approaches to solving formulations of the model. It includes data organization and input modules, solution presentation and initial test runs.

4.1. Data Organization and Input

The model requires two inputs - block layout and the loaded from-to flows. SPIRAL (Goetschalckx, 1992) uses the from-to flows and other adjacency considerations to generate a block layout. SPIRAL is a Microsoft Windows based program that displays the block layout. SPIRAL will be used as a front end to generate the block layout for this model. The block layout information as well as the stored loaded from-to flow information will be used to generate the MIP formulation of the model. Routines in SPIRAL will also be used to display the solution on the screen.

4.2. Solution Presentation

The inputs described in the previous section are used to generate the models. Preprocessing routines to convert the block layout to the node-arc representation as shown in Figure 2 have been completed. Solutions of the model as well as simple reports will be presented graphically. This will be accomplished by linking the solution generated to the SPIRAL graphical user interface. A sample screen is shown in Figure 3. The top left window shows the block layout and the other windows displays the solution of the formulation.

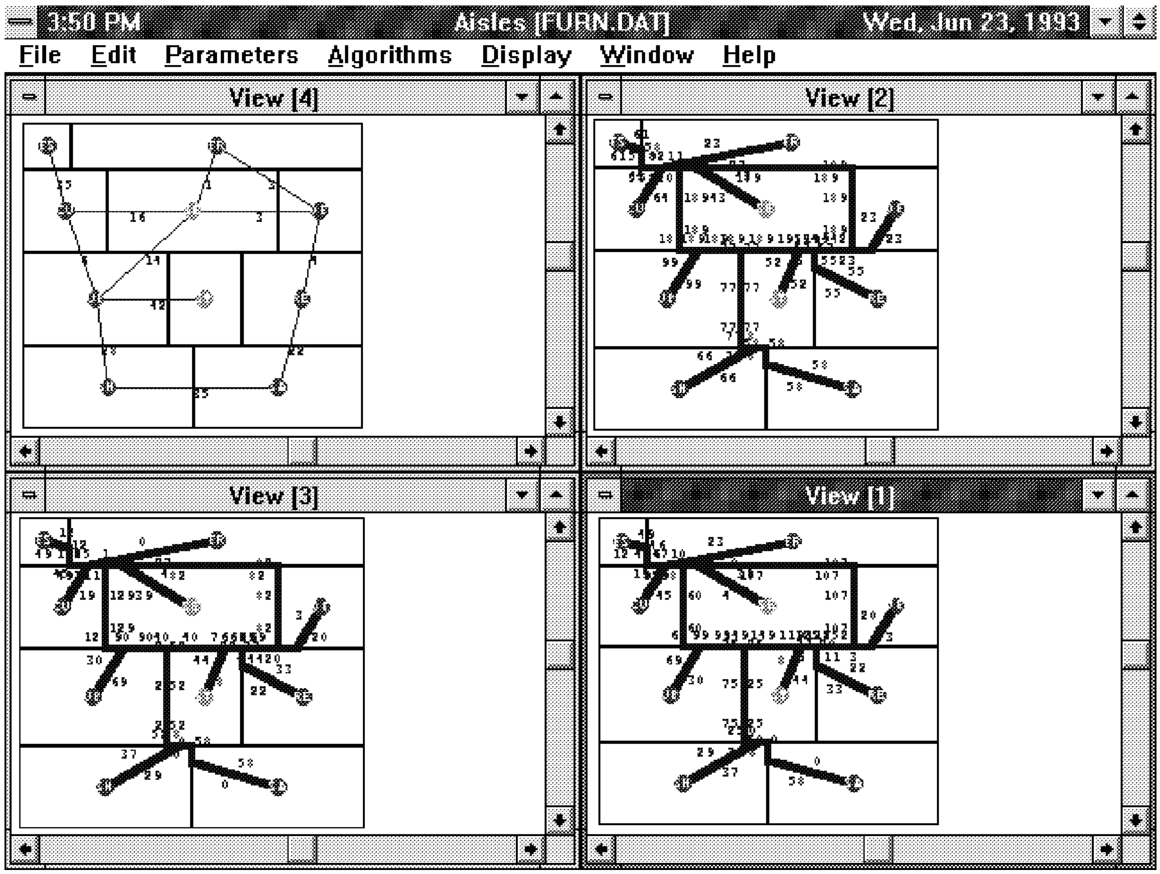

Figure 3 - An example block layout

4.3. Initial Test Runs

The data for the test cases are Nugent8, Nugent15 (Nugent, 1968) and Furn (Goetschalckx, 1992). The block layouts for these test cases were generated using SPIRAL. For the given block layout, the LP models for the two despatching strategies of unloaded flows were generated. The LP relaxation of the arc based models developed in section 3.7. were solved on a SUN SPARC machine using the CPLEX solver. Path based formulations of the problem were also generated to study it's solution convergence with that of the arc based model. Table 3 gives the results of the runs for the test cases.

Table 3 - LP runs for the test cases

Problem (# depts)	MC			Factor		
Solution	Constraints	Variables	Run time	Constraints	Variables	Run time
Nugent8 (8)						
Arc based						
Primal simplex	3750	2934	282 s	10752	8166	4238 s
Dual simplex	3750	2934	169 s	10752	8166	1837 s
Path based						
Primal simplex	10013	2013	872 s	42085	7054	5408 s
Dual simplex	10013	2013	302 s	42085	7054	2684 s
Furn (10)						
Arc based						
Primal simplex	8381	6404	1411 s	19872	26406	28379 s
Dual simplex	8381	6404	855 s	19872	26406	10638 s
Path based						
Primal simplex	24633	4548	4104 s	76550	13952	34972 s
Dual simplex	24633	4548	1015 s	76550	13952	13984 s
Nugent15 (15)						
Arc based						
Primal simplex	23076	16761	18283 s.	63792	24938	-
Dual simplex	23076	16761	6662 s.	63792	24938	-

A MIP formulation of the arc based formulation of Nugent15 problem was generated and an attempt was made to solve this using the MIP solver of CPLEX on the SUN SPARC machine. The problem ran for 300 CPU hours without finding the optimum solution. At this time a total of 20,000 nodes (system default) were generated and the solver aborted.

A construction heuristic was used to solve the MIP model formulation. Three criteria were used to decide the sequence of flows for opening the aisles and to assign the direction of aisles for the construction heuristic. These criteria were based on flow magnitude (H1), distance traveled (H2), and the scalar product of flow magnitude and distance traveled (H3). These heuristics were run on a 486/33 machine and the run times with the objective function values are given in Table 4.

Table 4 - Heuristic Solution Comparison

	H1		H2		H3		LP	
Problem	Objective	Time (s)	Objective	Time (s)	Objective	Time (s)	Objective	Time (s)
Furn								
MC	27243.37	1.27	27422.81	1.34	27422.81	1.34	23428.31	855
Factor	30462.13	17.53	30462.13	18.56	30877.15	18.61	27768.33	10638
Nugent15								
MC	28214.62	1.03	28156.58	7.31	28156.58	7.32	25741.29	6662
Factor	29647.28	68.84	31884.73	71.87	31884.73	71.87	-	-
Nugent20								
MC	40918.66	2.79	39957.81	26.84	39957.81	26.93	-	-
Factor	47753.48	252.47	46323.13	289.87	46323.13	288.50	-	-

4.4. Proposed Solution Strategy

Table 3 shows that the model size grows rapidly as the number of departments increases and so does the computing time, thus making it impractical to solve the model for larger data sets with standard MIP solvers using standard branch and bound based on LP relaxations. In future research the following approaches will be taken to solve the proposed model

- investigate reformulation of the model to identify special structures and exploit them,
- attempt to solve the original problem in a hierarchical manner; e.g., solve the sub problem of aisle selection first and then solve the station location and flow assignment,
- implement improvement heuristics in conjunction with H1, H2, and H3,
- determine bounds for the H1, H2, and H3 without having to solve the model using LP solvers.

5. CONCLUSIONS

Facilities design is an important problem faced by industrial engineers. An important component within the facilities design is the design of the material handling network. We present a model that determines simultaneously

- the location of the aisles
- the location of the P/D/C station
- the vehicle movement paths

- the vehicle dispatching (in some models) and
- the fleet sizing

when the transportation as well as the fixed setup costs are included in the model. Our model includes the correct unloaded flows and cost parameters useful in determining the material flow networks. The proposed model is more comprehensive than the existing models. Initial experiences in solving the proposed model are being reported. The material flow network along with the block layout can be used to perform a detailed layout of the facility (Montreuil, 1988) (Montreuil, 1990).

The solution of the model using standard methods for MIP solvers is impractical. Strategies to solve the model include reformulation and exploiting the problem structure, and heuristics.

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