

# Using Bucket Brigades to Migrate from Craft Manufacturing to Assembly Lines

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One way to organize workers that lies between traditional assembly lines, where workers are specialists, and craft assembly, where workers are generalists, are “bucket brigades.” We describe how one firm used bucket brigades as an intermediate strategy to migrate from craft assembly to assembly lines. The adoption of bucket brigades led to a narrowing of tasks for each worker and thus accelerated learning. The increased production more than compensated for the time lost when workers walk back to get more work, which was significant in this implementation.

To understand the trade-offs in migrating from craft to assembly lines, we extend the standard model of bucket brigades to capture hand-off and walk-back times.

*Key words:* bucket brigades; assembly line; work sharing; dynamical system; self-organizing system

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TUG™ is a company that assembles about 10 models of industrial tractor of the type commonly used at airports to pull luggage trains. Initially, the company was small and privately held but, when the founder retired, it was acquired by a professional management group whose goal was to grow the company aggressively. Indeed, the new owners had secured some large orders and, to meet sales commitments, had to increase production significantly and quickly.

Unfortunately, the way in which manufacturing was organized at TUG made it difficult to increase production. Each tractor was built by a single person (or, for the larger models, a two-person team) from start to finish. Assembly began after a fork-lift truck delivered an engine and a locker of parts to the work area and then placed a new tractor frame on four jack stands. Within a day or two, depending on the model, the tractor was assembled and driven off to the paint booth while work was begun on another.

Under this style of assembly, each worker had to

(1) know how to build a tractor from start to finish, and

(2) have a complete set of expensive tools.

Requirement (1) meant that it took months for a new employee to become proficient. TUG tried to hire aggressively, but requirement (2) narrowed the pool of available labor because each worker was expected to provide his own tools. Furthermore, under pressure to increase production, workers were pressed into so much overtime that the company suffered a 60% turnover rate. As a result, TUG faced a crisis: How to increase production quickly when it was so hard to train and retain workers?

TUG was trying to convert to a traditional assembly line in the hopes that specialization of labor would make it easier to train new workers as well as relax the tool requirement. TUG had worked for almost a year to build specialized jigs, hoists, conveyors, and other material-handling equipment for eventual conversion to an assembly line, but this process took some of their best labor away from assembly. To further complicate matters, there was still no formalization of task primitives, no work content models, and no work standards on which to base any sort of traditional assembly line. In addition, it was estimated that it would take months to document assembly of even the simplest tractor. TUG was daunted: They

had to increase production—but the only way forward seemed to be to shut down production for months while they documented work and reorganized assembly.

Within a few hours, we converted the manufacturing of their most popular tractor to a bucket brigade assembly line, which has previously been shown to be self-balancing (Bartholdi et al. 1999, 2001; Bartholdi and Eisenstein 1996, 1999). The self-balancing properties of bucket brigades had many useful consequences for TUG. First, because the line is self-balancing, formal task descriptions or work standards are unnecessary and so the bucket brigade line was easy to set up. Moreover, self-balancing narrowed the range of tasks required of each worker; this specialization resulted in an immediate increase in production. Furthermore, the self-balancing property of bucket brigades spontaneously reallocated work as new hires improved their skills. Finally, all this increased the retention of workers, and so TUG was able to enjoy the benefits of training its workforce.

We had to customize bucket brigades to work in the special circumstances at TUG, and this provided the occasion to extend the standard model of bucket brigades, on which all previous implementations had been based. In the following we describe the technical details of those changes, analyze their implications, and report performance at TUG; but the new idea here is a managerial one: that bucket brigades can serve as a means of smoothly migrating an organization from craft manufacturing to assembly line.

## 1. Bucket Brigades at TUG

Bucket brigades are a way of coordinating workers on an assembly line. As originally conceived, each worker moves from work station to work station progressively assembling a product. When the last worker finishes, he walks back and takes over the product of the worker immediately upstream, who similarly walks back to get more work, until the first worker returns to the starting work station and begins a new product. Workers remain in sequence because no passing is allowed. Elsewhere we have analyzed various models of bucket brigades and shown that under very general conditions, if workers are sequenced from slowest to fastest along the direction of material flow, the assembly line will

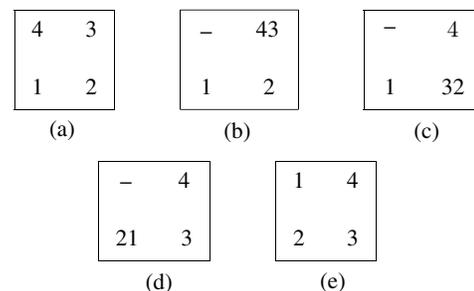
balance itself: That is, without management intervention, the workers will gravitate to an optimal division of the work and the production rate will be maximized (Bartholdi and Eisenstein 1996, 1999; Bartholdi et al. 2001, 2002).

To work in the TUG environment we had to adapt bucket brigades in some new and interesting ways. Most immediately, it was not possible to move tractors-in-process from work-station to work-station because this would have required expensive special purpose material-handling equipment. Consequently, we required that the workers move from tractor to tractor.

At TUG, four work stations are clustered around a shared crane. The movement of the four-worker bucket brigade is illustrated in Figure 1, where we have numbered the workers so that worker  $i$  is faster than worker  $i - 1$ . Each worker independently assembles a tractor. If, as is the normal state of affairs, worker  $i$ 's tractor is nearer completion than worker  $(i - 1)$ 's, then we would eventually observe the following sequence of events:

- Completion—Worker 4 finishes his tractor and it is driven off the assembly area;
- Walk-back—Worker 4 collects his tools and rolls his tool box to the tractor of worker 3;
- Hand-off—Workers 3 and 4 review and agree on what work remains to complete assembly of the tractor. When worker 4 understands exactly what

**Figure 1** Successive Snapshots of the Four-Worker Bucket Brigade at TUG



*Notes.* (a) Each of the four workers is assembling a tractor. (The work stations are clustered so that they share a crane.) (b) When 4, the fastest, finishes his tractor, it is driven to the paint booth and he pushes his tools to the station of his predecessor, 3, and takes over 3's tractor. (c) Worker 3 then moves back to take over 2's tractor. (d) Worker 2 in turn moves back to take over worker 1's tractor. (e) Worker 1, the slowest, starts a new tractor.

remains to be done, he assumes responsibility for the tractor and takes over its assembly;

- Remaining walk-backs/hand-offs—Worker 3 collects his tools, walks back to worker 2, and takes over assembly of his tractor, after which worker 2 goes back to take over assembly of the first tractor;
- Worker 1 moves his tools to the empty area vacated by the tractor newly completed by worker 4 and begins assembling a new tractor.

This process then repeats indefinitely, with each completion followed shortly by the start of a new tractor.

There are three significant differences in this behavior from that of the previous models of bucket brigades (Bartholdi et al. 1999, 2001, 2002; Bartholdi and Eisenstein 1996).

First, a slower worker may pass a faster worker. Because each worker has his own work station, the tractor of a slower worker may overtake—that is, advance closer to completion than—the tractor of a faster worker, if the latter is delayed for some reason.

Second, walk-back time is significant. At TUG, each worker must gather his tools and push his tool box to the assembly area of his predecessor, which takes three to five minutes.

Third, hand-off time is significant. The time to hand off work from one worker to another is significant. Both workers must wait until the downstream worker assumes responsibility for the tractor, which takes 10 to 20 minutes because the worker assuming responsibility must understand exactly what remains to be completed.

## 2. A Model of Bucket Brigades at TUG

Because walk-back and hand-off times are significant, and passing is allowed, we must extend the normative model of bucket brigades to understand the effects of these new issues. Key questions include: Will the bucket brigade retain the property of self-balance? How will the time lost to walk-back and hand-off change the balance? How will the lost time reduce the production rate?

### 2.1. Revised Bucket Brigade Rules for TUG

We begin by formalizing the revised bucket brigade rules that each TUG worker must follow. Let the workers be numbered from slowest to fastest  $1, \dots, n$ . Worker  $i$  is the predecessor of worker  $i + 1$ , and

worker  $i + 1$  is the successor of worker  $i$ . Under bucket brigades each worker follows these rules.

**Work forward:** Continue to assemble your item as quickly as possible. If you complete your item then **wait**; if you are preempted by your successor, hand off your work and **walk back** to get more work.

**Wait:** If you are the last worker (worker  $n$ ), remove your finished item from the assembly area, then **walk back** to get more work as soon as no other worker is **walking back**. If you are not the last worker, then wait until your successor takes over your item and then **walk back** to get more work.

**Walk back:** If you are the first worker (worker 1), walk back to the assembly area vacated by the last worker (worker  $n$ ) and begin assembling a new item. If you are not the first worker, walk back to the work station of your predecessor and take over assembly of his item. In either case begin to **work forward**.

Note that only the first, slowest worker is allowed to begin work on a new tractor, and only the last, fastest worker is allowed to initiate a walk-back. Furthermore, the last worker can begin a walk-back only when no other walk-back or hand-off is in progress. This differs from previous specifications of bucket brigade behavior, in all of which walk-backs and hand-offs were assumed to be instantaneous.

The waiting rule is also new. It restricts behavior so that no more than one worker at a time can walk back to get more work. This confers analytical tractability, because it prevents degenerate situations such as when worker  $i + 1$  walks back to take over the tractor of worker  $i$  but finds worker  $i$  in the process of taking over the tractor of worker  $i - 1$ . The waiting rule prevents this by requiring that any worker (except the last) who completes a tractor must wait until the fastest worker completes his tractor, and, if a walk-back or hand-off is in progress, the last worker must wait until the first worker begins a new tractor.

This has the potential of wasting productive effort because it requires some workers to stand idle. However, under normal operation of a properly configured bucket brigade line, the waiting rule will never be invoked, so the analytic tractability it confers costs us nothing in practice. Indeed, as we shall show later, invocation of the waiting rule means that there are too many workers in the bucket brigade, and so too many hand-offs.

Finally, we have stated the bucket brigade rules with more generality than one might think necessary, so that they continue to work even when things go wrong on the shop floor. For example, it is possible that one tractor may *pass* another; that is, the slower worker may advance assembly of his tractor beyond that of the faster worker. This could happen if the faster worker experienced some difficulty during assembly, such as inadvertently stripping a thread. Passing was forbidden in previous models of bucket brigade, but this was because of the context in which they operated, which included assembly along a sequence of work stations (Bartholdi and Eisenstein 1996) or order-picking in a warehouse (Bartholdi et al. 2001). Our revised rules ensure that, after passing, workers are returned to their proper sequence of slowest to fastest. We shall show that, even though our model of work content does not capture the possibility of random disruptions, the resultant behavior of the bucket brigade does indeed compensate for such events.

## 2.2. Adapting the Normative Model

To understand the behavior of bucket brigade assembly lines at TUG we here extend the normative model (Bartholdi et al. 1999, 2001). This model is a particularly simple one, consisting of three idealized conditions that guarantee the performance of bucket brigades. The simplicity of this model makes it a useful benchmark to guide implementations. A reasonable first approach is to try to make the work environment match the normative model as much as possible.

Two of the three fundamental assumptions of the normative model are reasonably satisfied by TUG:

**ASSUMPTION 1. SMOOTHNESS AND PREDICTABILITY OF WORK.** *The nominal work content of the product is a constant (which we normalize to 1), and continuous.*

This is reasonable whenever assembly consists of many tasks, none of which takes “too much” time compared with the total work content. Also implicit here is that each successive item is assembled in the same sequence, which was not strictly true at TUG but tended to be enforced by precedence constraints on assembly (for example, one must mount the axles before the wheels).

Under this assumption, we can represent the state of a partially completed tractor as a number between

0 and 1 giving the fraction of standard work-content that has been completed. (Note that bucket brigades do not require knowledge of either the standard work content or the fraction completed for any tractor.)

**ASSUMPTION 2. TOTAL ORDERING OF WORKERS BY VELOCITY.** *Each worker  $i = 1, \dots, n$  is characterized by a distinct, constant work velocity  $v_i$ .*

This remains, in our experience, common in assembly systems, including TUG, because of the constant economic pressure to simplify work and reduce variance. (Furthermore, it is shown in Bartholdi et al. 2001 that the essential behavior of bucket brigades remains unchanged even under models of stochastic work content.)

The remaining assumption of the normative model is the one we must relax in order to understand the performance of bucket brigades at sites such as TUG. In all previous applications we have assumed the following:

**ASSUMPTION 3. INSIGNIFICANT WALKING TIME.** *The total time to assemble a product is significantly greater than the time to walk the length of the flow line. Therefore all hand-offs occur, for all practical purposes, simultaneously, synchronized by item completions of the last worker.*

This was a questionable assumption at TUG, where it required as long as five minutes to pack up and move tools and 10 to 20 minutes to negotiate the hand-off of a partially completed tractor. With four workers engaged in the bucket brigade, there would be three hand-offs per tractor, each involving two workers, which represents one to two person-hours of lost production per tractor. The simplest model of tractor required about eight person-hours to assemble, so the production capacity lost due to walk-backs and hand-offs represented about 12%–25% of the total production capacity. It was not clear that bucket brigades could overcome such a disadvantage.

It is the relaxation of Assumption 3 that is the primary concern of this paper. To model the bucket brigade we implemented at TUG, we replace Assumption 3 with the following two assumptions, which describe the details of how work is transferred from one worker to another.

**ASSUMPTION 3.1. CONSTANT WALK-BACK TIMES.** *The time required for worker  $i$  to walk back to his predecessor is a constant  $b_i \geq 0$ .*

Assumption 3.1 differs from the normative model in that the time  $b_i$  to walk back can be nonzero; the time depends only on the identity of the worker. This made sense in the context of TUG because the work stations were at the corners of a square and so the walking distances were all identical, but some workers get their tools together and move to the previous work station faster than others.

**ASSUMPTION 3.2. CONSTANT HAND-OFF TIMES.** *After worker  $i$  has walked back to take over the work of his predecessor, the time required to negotiate the hand-off is a constant  $h_i \geq 0$ , after which worker  $i$  assumes work on the item for which he has just taken responsibility and worker  $i - 1$  begins walking back to take over the item of his predecessor.*

Assumption 3.2 does not specify how  $h_i$  is determined. Here are some possibilities that lie within our model—Consider a hand-off to consist of two subtasks: The upstream worker must relinquish the tractor, and the downstream worker must accept responsibility for the tractor. Let  $r_i$  be the time required by worker  $i$  to relinquish his tractor to the downstream worker; and let  $s_i$  be the time required by worker  $i$  to accept work from the upstream worker. Four plausible ways of determining hand-off time are  $h_i = \max\{r_{i-1}, s_i\}$  (the hand-off is complete only when both workers have agreed to the hand-off);  $h_i = s_i$  (the hand-off is complete only when worker  $i$ , the downstream worker, has assumed responsibility for the item);  $h_i = r_{i-1}$  (the hand-off is complete only when worker  $i - 1$ , the upstream worker, has relinquished the item); and  $h_i = \min\{r_{i-1}, s_i\}$  (the hand-off is complete as soon as either worker is satisfied).

Finally, it is worth remarking that we cannot simply sum the walk-back time  $b_i$  and hand-off time  $h_i$  into a single constant term because the state of the system evolves differently during walk-back times and hand-off times. For example, worker  $i - 1$  continues to make progress while worker  $i$  is walking back for duration  $b_i$ , but they both suspend work for duration  $h_i$ .

For notational convenience let  $B = \sum_{i=1}^n b_i$  be the total time required to perform all walk-backs (excluding hand-offs), and let  $H = \sum_{i=1}^n h_i$  be the total time required to perform all hand-offs between the time the last worker completes a product and the first

worker begins a new one. (Note that we take  $h_1 = 0$  because any setup time may be assumed to be part of the work content of the product.)

We shall find that, even with the waste of nonzero walk-back and hand-off times, a bucket brigade assembly line remains self-balancing. Furthermore, even though it loses some production capacity in theory (the amount of which we shall make precise in what follows), we were surprised to find that in practice bucket brigades were nevertheless an improvement.

### 3. Dynamics of the TUG Assembly Line

In our model, all tractors share a common work content, so the state of the system at any time may be summarized by giving, for each worker, the fraction of work content that has been completed. As in Bartholdi and Eisenstein (1996), we refer to the fraction completed by a worker as his position and we restrict our attention to the sequence of worker positions  $\{\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(k)}, \dots\}$  at those instants when a new tractor is begun by the first worker. The vector  $\mathbf{x}^{(k)}$  is the  $k$ th iterate of the bucket brigade system. Because our rules mandate that no worker is walking back or preempting whenever a new tractor is begun, then all workers have an item at the start of an iteration, and, therefore, the worker positions  $x_1^{(k)}, x_2^{(k)}, \dots, x_n^{(k)}$  are well-defined over the work content  $[0, 1]$  for all  $k$ . Let  $f$  be the function that maps the vector of worker positions at the start of one iteration to that at the subsequent iteration, so that  $\mathbf{x}^{(k+1)} = f(\mathbf{x}^{(k)})$ . We study the behavior of a bucket brigade line by studying its trajectory  $\{\mathbf{x}^{(k)} = f^k(\mathbf{x}^{(0)})\}_{k=0}^{\infty}$ .

The function  $f$  expresses the following dynamics. An iteration begins when the first worker begins a new tractor, and thus  $x_1^{(k+1)} = 0$ . During an iteration, worker  $i - 1$  progresses at rate  $v_{i-1}$  for time  $(1 - x_n^{(k)})/v_n$ , until worker  $n$  completes his tractor. Then, unlike behavior in previous models of bucket brigades, worker  $i - 1$  continues at rate  $v_{i-1}$  until he is preempted; this will require time  $\sum_{j=i}^n b_j + \sum_{j=i+1}^n h_j$ . After worker  $i - 1$  is preempted by worker  $i$ , worker  $i$

works at rate  $v_i$  while worker  $i - 1$  walks back and preempts worker  $i - 2$ , and so forth, until worker 1 begins a new tractor; this requires time  $\sum_{j=1}^{i-1} b_j + \sum_{j=2}^{i-1} h_j$ . However, the forward progress of a worker may be restricted by the end of the line, in which case he waits at position 1 until his item is taken over by his successor. These dynamics are summarized by:

$$\begin{aligned} x_1^{(k+1)} &= 0, \\ x_i^{(k+1)} &= \min \left\{ 1, x_{i-1}^{(k)} + (1 - x_n^{(k)}) \frac{v_{i-1}}{v_n} \right. \\ &\quad \left. + \left( \sum_{j=i}^n b_j + \sum_{j=i+1}^n h_j \right) v_{i-1} + \left( \sum_{j=1}^{i-1} b_j + \sum_{j=2}^{i-1} h_j \right) v_i \right\} \\ &\quad \text{for } i=2, \dots, n. \end{aligned} \quad (3.1)$$

### 3.1. Balance

An assembly line is *balanced* if each worker repeats the same interval of work content on successive items. For a bucket brigade, this means that there exists a fixed point  $\mathbf{x}^* = f(\mathbf{x}^*)$ , such that, if the workers begin at these positions, then, after completion of each item, they will walk back to exactly these same positions to begin work on the subsequent item.

From Equations (3.1), we can express the fixed point position  $x_i^*$  of each worker in terms of  $x_n^*$  to get,

$$\begin{aligned} x_1^* &= 0, \\ x_i^* &= \min \left\{ 1, \frac{1 - x_n^*}{v_n} \sum_{j=1}^{i-1} v_j + \sum_{j=1}^{i-1} (\bar{B}_j + \bar{H}_j) v_j + v_i \sum_{j=1}^{i-1} (b_j + h_j) \right\} \\ &\quad \text{for } i=2, \dots, n, \end{aligned} \quad (3.2)$$

where we define  $\bar{B}_i = B - b_i$ , and  $\bar{H}_i = H - h_{i+1} - h_i$ , and for notational convenience we define  $h_{n+1} = 0$ . Solving for  $x_n^*$  yields

$$x_n^* = \min \left\{ 1, \frac{\sum_{i=1}^{n-1} v_i + v_n \sum_{i=1}^n (\bar{B}_i + \bar{H}_i) v_i}{\sum_{i=1}^n v_i} \right\}. \quad (3.3)$$

We now have the following property for any fixed point.

LEMMA 3.1. For any fixed point  $\mathbf{x}^*$ , the workers are in positional sequence, so that  $0 = x_1^* \leq x_2^* \leq \dots \leq x_n^* \leq 1$ .

PROOF. Follows from Equations (3.2).  $\square$

The following lemma establishes that the fixed point for our system is unique:

LEMMA 3.2. There is a unique fixed point  $\mathbf{x}^*$  to the system of Equations (3.1).

PROOF. Equation (3.3) returns a unique value for  $x_n^*$ , which is then substituted into Equations (3.2).  $\square$

The time  $B + H$  that is lost in walk-back and hand-off during assembly of each tractor increases with the number of workers in a bucket brigade. In fact, if there are too many workers, then the wasted person-hours could exceed the time required to assemble a tractor. In such case, the waiting rule would keep some workers permanently idle, waiting at 1 for the current hand-off to ripple upstream. In the extreme case of many workers, the fixed point could degenerate to  $\mathbf{x}^* = (0, 1, \dots, 1)$  so that only the first and possibly second workers contribute to production.

We say that the bucket brigade is *overstaffed* if  $x_n^* = 1$ , for then the efforts of at least one worker are completely forfeit because the accumulated waste exceeds the work content of the product. The implication for practice is clear: One must not overstaff the bucket brigade.

Equation (3.3) gives the precise condition necessary for  $x_n^* < 1$ . We note that  $x_n^* < 1$  only if the term  $\sum_{i=1}^n (\bar{B}_i + \bar{H}_i) v_i < 1$ . This term accumulates the total amount of work accomplished over all workers during hand-offs and walk-backs. Thus, if a unit can be completed during the time required to complete the series of walk-backs and hand-offs, then the line is overstaffed, or, roughly speaking, the bucket brigade is overstaffed if the unproductive time exceeds the productive time. The following lemma provides a simple, sufficient condition for a line not to be overstaffed. This can be helpful as a managerial first check.

LEMMA 3.3. The bucket brigade is not overstaffed (that is,  $x_n^* < 1$ ) if  $B + H < 1 / ((n - 1) v_{\max})$ .

PROOF. From Equation (3.3),  $x_n^* < 1$  as long as  $\sum_{i=1}^n (\bar{B}_i + \bar{H}_i) v_i < 1$ , so the lemma holds because  $\sum_{i=1}^n (\bar{B}_i + \bar{H}_i) v_i < (B + H)(n - 1) v_{\max}$ .  $\square$

We determined that overstaffing would not be a problem at TUG, where four workers assembled its most popular tractor. The TUG assembly line would not be overstaffed until there were about 17 workers, which would mean 16 walk-backs and hand-offs per tractor, each involving 2 workers for about 15 minutes, when the total waste would match the 8 person-hours of work content.

### 3.2. Self-Balance

If the workers are sequenced from slowest to fastest, then all trajectories converge to the unique fixed point.

**THEOREM 3.4.** *If the workers are sequenced from slowest to fastest,  $v_1 \leq v_2 \leq v_3 \leq \dots \leq v_{n-1} < v_n$ , then any trajectory of worker positions  $\{\mathbf{x}^{(k)} = f^k(\mathbf{x}^{(0)})\}_{k=0}^\infty$  converges to the unique fixed point.*

**PROOF.** See the appendix.  $\square$

### 3.3. Production Rate

The following lemma gives the production rate of the bucket brigade at the fixed point.

**LEMMA 3.5.** *If the line is overstaffed (that is, if  $x_n^* = 1$ ), then the production rate at the fixed point is*

$$\frac{1}{B + H}.$$

*If the line is not overstaffed (that is, if  $x_n^* < 1$ ), the production rate at the fixed point is*

$$\frac{\sum_{i=1}^n v_i}{1 + \sum_{i=1}^n v_i(b_i + h_i + h_{i+1})}. \quad (3.4)$$

**PROOF.** We let  $t^*$  denote the cycle time, or time between successive item completions, when the system is at its fixed point. Thus  $t^* = (1 - x_n^*)/v_n + B + H$ . If  $x_n^* < 1$ , then from Equation (3.3) we get  $t^* = (1 + \sum_{i=1}^n v_i(b_i + h_i + h_{i+1}))/\sum_{i=1}^n v_i$ .  $\square$

Lemma 3.5 shows that, when the line is overstaffed (equivalently, when the time to walk back and hand off the product is very large), the production rate is determined entirely by the material handling time (walk-back and hand-off).

When the line is not overstaffed, the production rate is given by Equation (3.4). Thus, if walk-back and hand-off times were negligible, then, as would be expected, the production rate is simply the sum of the worker velocities. Otherwise, we interpret  $b_i + h_i + h_{i+1}$  as the unproductive time worker  $i$  must incur for each unit produced, and thus  $v_i(b_i + h_i + h_{i+1})$  is his lost work. Therefore, Equation (3.4) can be interpreted as the total productive capacity divided

by the total work content required (one unit of productive work plus  $\sum_{i=1}^n v_i(b_i + h_i + h_{i+1})$  units of unproductive work).

We also note that the first and last workers are somewhat underrepresented in the denominator of Expression (3.4). This is because  $h_1 = h_{n+1} = 0$ ; the first worker and last worker are each involved in a single hand-off, whereas all other workers participate in two. This is more easily seen by considering the simple case where all walk-back times are equal,  $b_i = b$ , and all hand-off times are equal,  $h_i = h$ . In this case the production rate simplifies to

$$\frac{1}{[1 - h(v_1 + v_n)]/\sum_{i=1}^n v_i + 2h + b}. \quad (3.5)$$

The terms  $v_1$  and  $v_n$  are represented in an extra term. This reflects the fact that the first (slowest) and last (fastest) workers contribute more of their time to production than do the other workers.

This suggests additional ways that one might try to increase production rate of a bucket brigade. For example, increases in the work velocities of the first and last worker are more valuable than increases in the velocities of any other workers. Similarly, Expression (3.4) shows that the greatest opportunity for improving the production rate lies in reducing the time to walk back and hand off work for the fastest workers.

## 4. Bucket Brigades Enable Rapid Learning

Under bucket brigades, the workers at TUG lost time in walk-backs and hand-offs and still increased the production rate by about 10% in the first week. We believe that this was due to increased rate of learning among workers, which was made possible by the specialization that emerged as the line balanced itself.

Imagine a single new worker joining three experienced workers, each producing a tractor with eight standard person-hours of work-content. If the new worker performed at half the speed of a seasoned worker, under craft-manufacturing he would complete one tractor every two days. But if the four workers were organized as a bucket brigade, the new worker would repeat the same initial portion of work-content at least three times a day. This frequent repetition is the basis for the faster learning. In addi-

tion, because of the self-balancing nature of bucket brigades, the new worker will spontaneously assume a larger portion of work content as his skills increase.

Given enough time, the workers at TUG could have become even more productive under craft manufacturing than under bucket brigades because of the time lost under bucket brigades to walk-backs and hand-offs—but the urgent need to respond to customer orders did not allow TUG the luxury of waiting. The production rate had to be increased as soon as possible.

The high productivity of bucket brigades under learning is consistent with recent work of Villalobos and Munoz (1999) and Munoz and Villalobos (2002), who concluded from simulation studies that bucket brigades outperform alternative ways of organizing workers when there is high labor turnover. (In related work, Armbruster and Gel 2002 used simulation to study the detailed dynamics of bucket brigades under some models of learning.)

## 5. Conclusions

Bucket brigades can be viewed as a way to organize workers; it lies between traditional assembly, where workers are specialists in fixed subintervals of work content, and craft assembly, where workers are generalists. At TUG, bucket brigades provided a way for the organization to evolve gracefully from craft assembly toward assembly lines to gain worker specialization and consequent learning. Importantly, TUG could take this step without sacrificing production rates. This gave TUG time to build a work-content model and plan an assembly line. (Indeed, the bucket brigade may be imagined to do some of the planning itself: Because the bucket brigade is self-balancing, the emergent allocation of work is strongly suggestive of how work should be distributed on the eventual assembly line.)

The success of this implementation depended on one insight and one piece of serendipity. At first it did not seem to us that bucket brigades could be used at TUG because the tractors had to remain stationary while under construction. It was surprisingly difficult to see what is now obvious, that the workers could rotate among the work stations. However even then we feared that bucket brigades would be less productive than craft assembly due to the considerable time

lost to walk-backs and hand-offs. Fortunately—and, to us, surprisingly—this was more than made up for by increased learning.

TUG eventually did move to an assembly-line but this was for a mix of reasons, none having to do with the technical aspects of bucket brigades. Most immediately, there was a complete change in management after TUG was acquired by a larger company, Stewart & Stevenson Services, Inc., which has particular strengths in paced assembly. That company invested the engineering and capital required to convert entirely to traditional assembly lines. It remains possible for Stewart & Stevenson to revert to bucket brigades, especially now that the work content has been carefully documented, making hand-offs more efficient.

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## Appendix. Proof of Theorem 3.4: Convergence to the Fixed Point

We introduce a new coordinate system that measures the time allocation between worker  $i$  and  $i + 1$  at the start of iteration  $k$ :

$$t_i^{(k)} = \frac{x_{i+1}^{(k)} - x_i^{(k)} - \left(\sum_{j=1}^i b_j + \sum_{j=2}^i h_j\right)v_{i+1}}{v_i} + \sum_{j=1}^i b_j + \sum_{j=2}^i h_j \quad \text{for } i = 1, \dots, n-1$$

$$t_n^{(k)} = \frac{1 - x_n^{(k)}}{v_n} + B + P.$$

Combining this coordinate system with Equations (3.1) yields

$$x_i^{(k+1)} = x_{i-1}^{(k)} + \left(t_n^{(k)} - B - P\right)v_{i-1} + \left(\sum_{j=i}^n b_j + \sum_{j=i+1}^n h_j\right)v_{i-1} + \left(\sum_{j=1}^{i-1} b_j + \sum_{j=2}^{i-1} h_j\right)v_i$$

$$= x_{i-1}^{(k)} + \left(t_n^{(k)} - \sum_{j=1}^{i-1} b_j - \sum_{j=1}^{i-1} h_j\right)v_{i-1} + \left(\sum_{j=1}^{i-1} b_j + \sum_{j=2}^{i-1} h_j\right)v_i.$$

Now we can rewrite the time allocation as

$$\begin{aligned}
 t_i^{(k+1)} &= \frac{1}{v_i} \left[ x_i^{(k)} + \left( t_n^{(k)} - \sum_{j=1}^i b_j - \sum_{j=1}^{i+1} h_j \right) v_i \right. \\
 &\quad \left. + \left( \sum_{j=1}^i b_j + \sum_{j=2}^i h_j \right) v_{i+1} \right] \\
 &\quad - \frac{1}{v_i} \left[ x_{i-1}^{(k)} + \left( t_n^{(k)} - \sum_{j=1}^{i-1} b_j - \sum_{j=1}^i h_j \right) v_{i-1} \right. \\
 &\quad \left. + \left( \sum_{j=1}^{i-1} b_j + \sum_{j=2}^{i-1} h_j \right) v_i \right] \\
 &\quad - \frac{\left( \sum_{j=1}^i b_j + \sum_{j=2}^i h_j \right) v_{i+1}}{v_i} + \sum_{j=1}^i b_j + \sum_{j=2}^i h_j \\
 &= \frac{x_i^{(k)} - x_{i-1}^{(k)} - \left( \sum_{j=1}^{i-1} b_j + \sum_{j=2}^{i-1} h_j \right) v_i}{v_i} \\
 &\quad + \frac{\left( \sum_{j=1}^{i-1} b_j - \sum_{j=1}^i h_j \right) v_{i-1}}{v_i} + (1 - v_{i-1}/v_i) t_n^{(k)} \\
 &= (v_{i-1}/v_i) t_{i-1}^{(k)} + (1 - v_{i-1}/v_i) t_n^{(k)}.
 \end{aligned}$$

And, thus, rewriting these equations as a linear system

$$\mathbf{t}^{(k+1)} = T \mathbf{t}^{(k)},$$

we observe that when  $v_1 \leq v_2 \leq \dots \leq v_{n-1} < v_n$ , the sequence of iterates converges, because  $T$  is the transition matrix of a finite state Markov chain that is irreducible and aperiodic.

Now, if at some iterations workers are held at the end of the line (and thus their  $x_i$ 's are dampened to 1), then the transition matrix will be substochastic. Because the cycle times are bounded below, however, the system converges to the fixed point.

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