

# ISYE 8843 Final Exam

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1. Bayesian Wavelet Shrinkage. This open ended question is essentially asking to select a data set with noise present in it (a noisy signal, function, or noisy image), transform the data to the wavelet domain, apply shrinkage by suitably developed Bayes rules on wavelet coefficients, and back-transform shrunk coefficients (alias Bayes estimates) to the domain of original data. Recent Tech Report 34/2004 at <http://www.isye.gatech.edu/brani/isyestat/> updates the Handout 21 with some recent references, and you may use some of the procedures supplied there. However, the question is open ended and you may propose your own method and use the software of your choice, even BUGS.

Solution:

Suppose the likelihood for a detail wavelet coefficient is given  $\theta$  is binomial  $\mathcal{B}(n, \theta)$ , i.e.,

$$f(x|\theta) = \binom{n}{x} \theta^x (1 - \theta)^{n-x}, \quad x = 0, 1, \dots, n,$$

and the prior is  $\beta, \mathcal{B}(\alpha, \beta)$ , where the hyperparameters  $\alpha$  and  $\beta$  are known,

$$\pi(\theta) = \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1 - \theta)^{\beta-1}, \quad 0 \leq \theta \leq 1.$$

The Bayesian rule is

$$\frac{x + \alpha}{n + \alpha + \beta}.$$

Take Doppler as the original signal, as known in Figure 1. Noise of size of 0.1 is added to the original signal. Figure 2 shows the data with noise adding to it. Figure 3 shows the shrinkage results by applying the Bayesian rule above to the data in Figure 2.

The codes for this problem is in the Appendix.

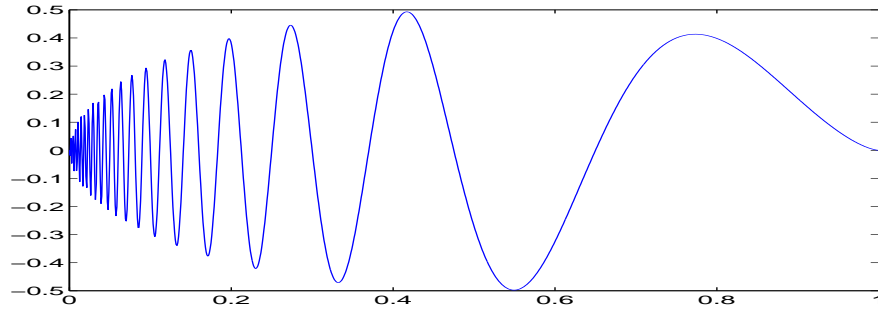


Figure 1: Original Data

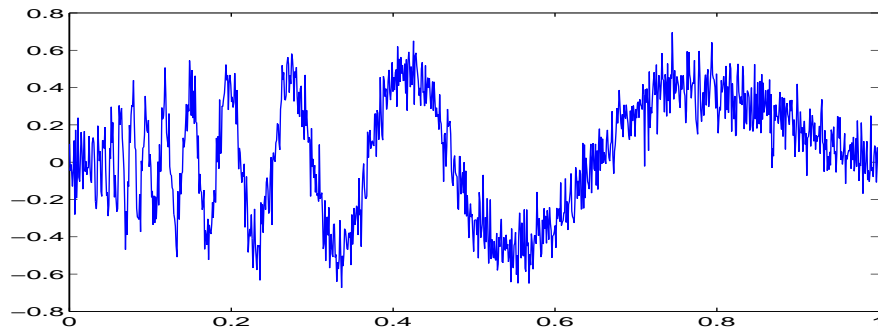


Figure 2: Original Data with Noise

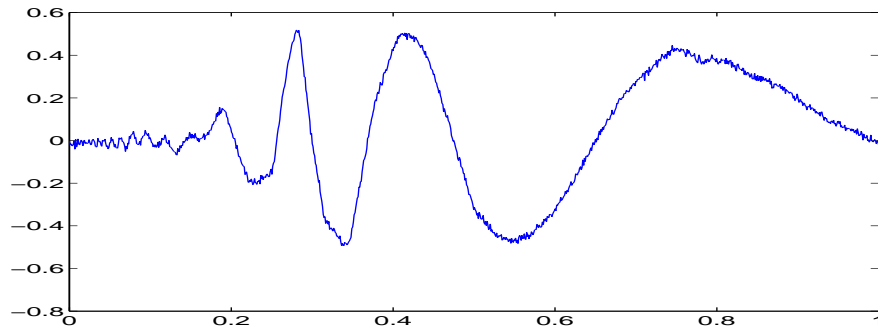


Figure 3: Results of Shrinkage

**Problem 2 Solution:**

(i) Exact calculating:

$$\begin{aligned}P(J1, M1|B1) &= \frac{1}{P(B1)} \sum_{E,A} P(B1, E, A, J1, M1) \\&= \frac{1}{P(B1)} \sum_A \left[ \sum_E P(E)P(A|B1, E) \right] P(B1)P(M1|A)P(J1|A) \\&= \frac{1}{P(B1)} \sum_A P(A, B1)P(B1)P(M1|A)P(J1|A)\end{aligned}$$

With

$$\begin{aligned}P(A0, B1) &= P(E0)P(A0|B1, E0) + P(E1)P(A0|B1, E1) \\&= 0.998 * 0.06 + 0.002 * 0.05 \\&= 0.05998,\end{aligned}$$

and

$$\begin{aligned}P(A1, B1) &= P(E0)P(A1|B1, E0) + P(E1)P(A1|B1, E1) \\&= 0.998 * 0.94 + 0.002 * 0.95 \\&= 0.94002.\end{aligned}$$

Then,

$$\begin{aligned}P(J1, M1|B1) &= P(A0, B1)P(M1|A0)P(J1|A0) + P(A1, B1)P(M1|A1)P(J1|A1) \\&= 0.05998 * 0.01 * 0.05 + 0.94002 * 0.70 * 0.90 \\&= 0.5922.\end{aligned}$$

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(ii) Using Kevin Murphy's BNT

With

$$P(J1|M1, B1) = 0.8992,$$

$$P(M1|B1) = 0.6586,$$

the results is:

$$P(J1, M1|B1) = P(J1|M1, B1)P(M1|B1) = 0.8992 * 0.6586 = 0.5922.$$

See Appendix for codes.

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(iii) Using BUGS.

With

$$P(J1|M1, B1) = 0.897,$$

$$P(M1|B1) = 0.649,$$

the results is:

$$P(J1, M1|B1) = P(J1|M1, B1)P(M1|B1) = 0.897 * 0.649 = 0.5822.$$

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**Problem 3 Solution::**

- Full conditionals:

$$\begin{aligned}
 f(Y, \mu, \lambda, \tau) &= \left\{ \prod_{i=1}^{\tau} \mathcal{Poi}(\lambda) \right\} \pi(\tau) \pi(\lambda) \left\{ \prod_{i=\tau+1}^n \mathcal{Poi}(\mu) \right\} \pi(\tau) \pi(\mu) \\
 &= \frac{\lambda^{\sum_{i=1}^{\tau} y_i} e^{-\tau\lambda} \lambda^{\alpha_\lambda - 1} e^{-\beta_\lambda \lambda}}{(\prod_{i=1}^{\tau} y_i!) n \Gamma(\alpha_\lambda) \beta_\lambda^{\alpha_\lambda}} \frac{\lambda^{\sum_{i=\tau+1}^n y_i} e^{-(n-\tau)\mu} \mu^{\alpha_\mu - 1} e^{-\beta_\mu \mu}}{n (\prod_{i=\tau+1}^n y_i!) \Gamma(\alpha_\mu) \beta_\mu^{\alpha_\mu}} \\
 &= \frac{\lambda^{\sum_{i=1}^{\tau} y_i + \alpha_\lambda - 1} e^{-\lambda(\tau + \beta_\lambda)}}{(\prod_{i=1}^{\tau} y_i!) n \Gamma(\alpha_\lambda) \beta_\lambda^{\alpha_\lambda}} \frac{\mu^{\sum_{i=\tau+1}^n y_i + \alpha_\mu - 1} e^{-\mu(n - \tau + \beta_\mu)}}{(\prod_{i=\tau+1}^n y_i!) n \Gamma(\alpha_\mu) \beta_\mu^{\alpha_\mu}}
 \end{aligned}$$

From the above equation,

$$P(\tau = k | \lambda, \mu, Y) = \lambda^{\sum_{i=1}^{\tau} y_i + \alpha_\lambda - 1} e^{-\lambda(\tau + \beta_\lambda)} \mu^{\sum_{i=\tau+1}^n y_i + \alpha_\mu - 1} e^{-\mu(\tau + \beta_\mu)},$$

$$\begin{aligned}
 [\lambda | \tau, \mu, Y] &= \frac{\lambda^{\sum_{i=1}^{\tau} y_i + \alpha_\lambda - 1} e^{-\lambda(\tau + \beta_\lambda)}}{\Gamma(\sum_{i=1}^{\tau} y_i + \alpha_\lambda) \beta_\lambda^{\sum_{i=1}^{\tau} y_i + \alpha_\lambda}}, \text{ and} \\
 [\mu | \tau, \lambda, Y] &= \frac{\mu^{\sum_{i=\tau+1}^n y_i + \alpha_\mu - 1} e^{-\mu(\tau + \beta_\mu)}}{\Gamma(\sum_{i=\tau+1}^n y_i + \alpha_\mu) \beta_\mu^{\sum_{i=\tau+1}^n y_i + \alpha_\mu}}.
 \end{aligned}$$

- Figure 4 shows the observations. Figure 5 shows the last 500 simulations out of 10000 for  $\tau$ ,  $\lambda$ , and  $\mu$ . Figure 6 shows the histograms for  $\tau$ ,  $\lambda$ , and  $\mu$ . Figure 7 shows the original data with the posteriors of  $\tau$ ,  $\lambda$ , and  $\mu$ .
- Table 1 shows the posterior sample means and variances for  $\tau$ ,  $\lambda$ , and  $\mu$

See Appendix A for codes.

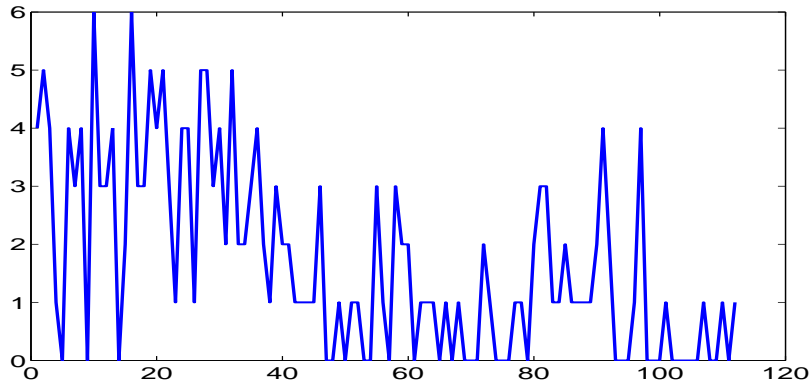


Figure 4: Original Data

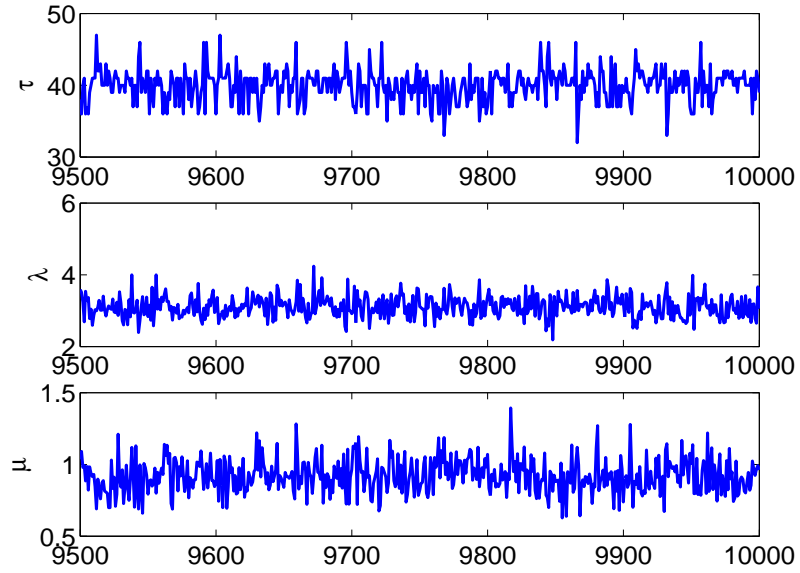


Figure 5: Last 500 Simulations for  $\tau$ ,  $\lambda$ , and  $\mu$

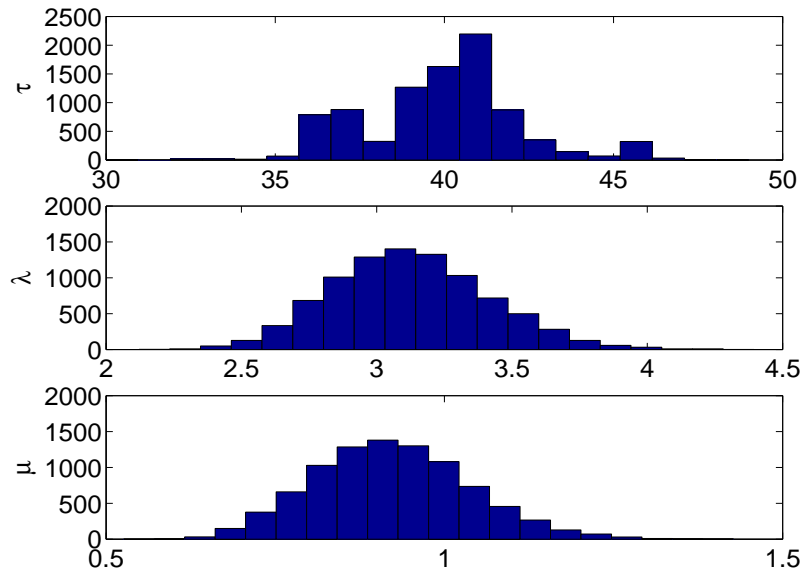


Figure 6: Histograms of  $\tau$ ,  $\lambda$ , and  $\mu$

Table 1: Posterior Statistics

Parameter	Mean	Variance
$\tau$	39.9681	6.0111
$\lambda$	3.1231	0.0850
$\mu$	0.9222	0.0133

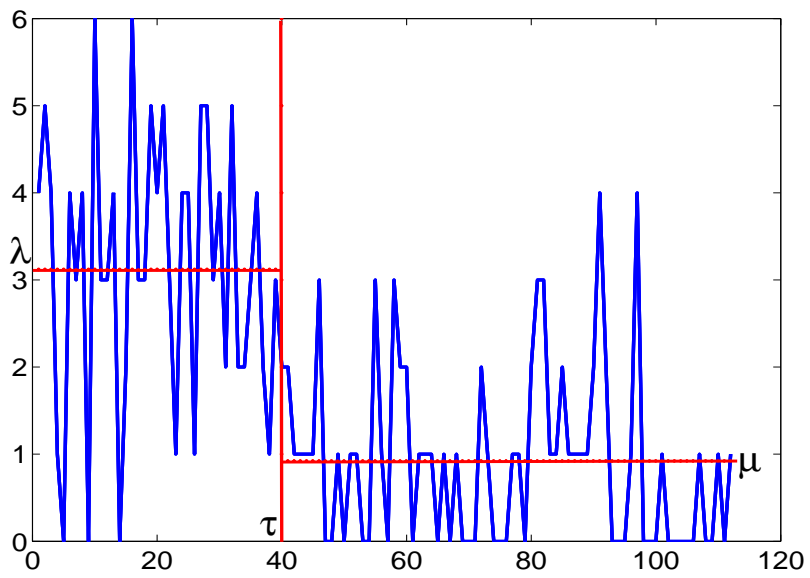


Figure 7: Original Data with Posteriors of  $\tau$ ,  $\lambda$ , and  $\mu$