MIDTERM EXAM (ISYE8843 FALL 2004)

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1. SOLUTION TO PROBLEM 1

- the matlab code for this problem.

```
clear all ;
close all ;

y1 = [25.8 19.8 28.6 29.4 22.3 33.8 33.8 27.8 29.6] ;
y2 = [16.5 23.5 13.5 34.6 16.9 18.8 26.1 18.4 17.2 11.6 20.2] ;
y3 = [24.0 29.1 16.0 24.8 27.0 10.9 11.8 23.2 17.7 23.9 24.6 24.0 27.2 23.7] ;

% set random generator seed
randn('seed',1)

y1_mean = mean(y1) ;
n1      = 9 ;
s1_2    = var(y1) ;

y2_mean = mean(y2) ;
n2      = 11 ;
s2_2    = var(y2) ;

y3_mean = mean(y3) ;
n3      = 14 ;
s3_2    = var(y3) ;

% set the parameters to be simulated
Mu_1 = [] ;
Mu_2 = [] ;
Mu_3 = [] ;
Psi = [] ;
Tau2 = [] ;
Sigma2 = [] ;
Sigma1_2 = [] ;
Sigma2_2 = [] ;
Sigma3_2 = [] ;

% set the parameters
k      = 3 ;
```

```
a0    = 1 ;
c0    = 1 ;
d0    = 1 ;
f0    = 1 ;
g0    = 0.1 ;
psi0  = 10 ;
zeta0 = 0.1 ;

M = 5000 ;
burn = 1000 ;

% generate initial values
tau2     = 1 / rand_gamma(c0/2, d0/2) ;
sigma2   = rand_gamma(f0/2, g0/2) ;
sigma1_2 = 1 / rand_gamma(a0/2, a0 * sigma2 / 2) ;
sigma2_2 = 1 / rand_gamma(a0/2, a0 * sigma2 / 2) ;
sigma3_2 = 1 / rand_gamma(a0/2, a0 * sigma2 / 2) ;
psi      = randn(1,1) * sqrt(tau2/zeta0) + psi0 ;

% Starting simulation
for ii = 1 : M
    mu1 = randn(1,1) * sqrt(1/(n1/sigma1_2 + 1/tau2)) + ...
    (n1 * y1_mean/sigma1_2 + psi/tau2)/(n1/sigma1_2 + 1/tau2) ;
    mu2 = randn(1,1) * sqrt(1/(n2/sigma2_2 + 1/tau2)) + ...
    (n2 * y2_mean/sigma2_2 + psi/tau2)/(n2/sigma2_2 + 1/tau2) ;
    mu3 = randn(1,1) * sqrt(1/(n3/sigma3_2 + 1/tau2)) + ...
    (n3 * y3_mean/sigma3_2 + psi/tau2)/(n3/sigma3_2 + 1/tau2) ;
    psi = randn(1,1) * sqrt(tau2/(k + zeta0)) + ...
    (mu1 + mu2 + mu3 + zeta0 * psi0)/(k + zeta0);
    tau2 = 1 / rand_gamma((c0 + k + 1)/2, ...
    (d0 + (mu1 - psi)^2 + (mu2 - psi)^2 + ...
    (mu3 - psi)^2 + zeta0 * (psi - psi0)^2)/2) ;
    sigma1_2 = 1 / rand_gamma((a0 + n1)/2, ...
    (a0 * sigma2 + (n1 - 1)*s1_2 + n1*(y1_mean - mu1)^2)/2) ;
    sigma2_2 = 1 / rand_gamma((a0 + n2)/2, ...
    (a0 * sigma2 + (n2 - 1)*s2_2 + n2*(y2_mean - mu2)^2)/2) ;
    sigma3_2 = 1 / rand_gamma((a0 + n3)/2, ...
    (a0 * sigma2 + (n3 - 1)*s3_2 + n3*(y3_mean - mu3)^2)/2) ;
    sigma2    = rand_gamma((f0 + k*a0)/2, ...
    (g0 + a0/sigma1_2 + a0/ sigma2_2 + a0/sigma3_2)/2) ;

    Mu_1 = [Mu_1 mu1] ;
    Mu_2 = [Mu_2 mu2] ;
    Mu_3 = [Mu_3 mu3] ;
    Psi  = [Psi psi] ;
    Tau2 = [Tau2 tau2] ;
    Sigma2 = [Sigma2 sigma2] ;
    Sigma1_2 = [Sigma1_2 sigma1_2] ;
    Sigma2_2 = [Sigma2_2 sigma2_2] ;
    Sigma3_2 = [Sigma3_2 sigma3_2] ;
end

% compute the mean and variance for each parameters
```

```
mu1mean = mean(Mu_1(burn+1:M)) ;
mu1var  = var(Mu_1(burn+1:M)) ;

mu2mean = mean(Mu_2(burn+1:M)) ;
mu2var  = var(Mu_2(burn+1:M)) ;

mu3mean = mean(Mu_3(burn+1:M)) ;
mu3var  = var(Mu_3(burn+1:M)) ;

sigma1_2_mean = mean(Sigma1_2(burn+1:M)) ;
sigma1_2_var  = var(Sigma1_2(burn+1:M)) ;

sigma2_2_mean = mean(Sigma2_2(burn+1:M)) ;
sigma2_2_var  = var(Sigma2_2(burn+1:M)) ;

sigma3_2_mean = mean(Sigma3_2(burn+1:M)) ;
sigma3_2_var  = var(Sigma3_2(burn+1:M)) ;

sigma2mean = mean(Sigma2(burn+1:M)) ;
sigma2var  = var(Sigma2(burn+1:M)) ;

tau2_mean = mean(Tau2(burn+1:M)) ;
tau2_var  = var(Tau2(burn+1:M)) ;

psi_mean = mean(Psi(burn+1:M)) ;
psi_var  = var(Psi(burn+1:M)) ;
```

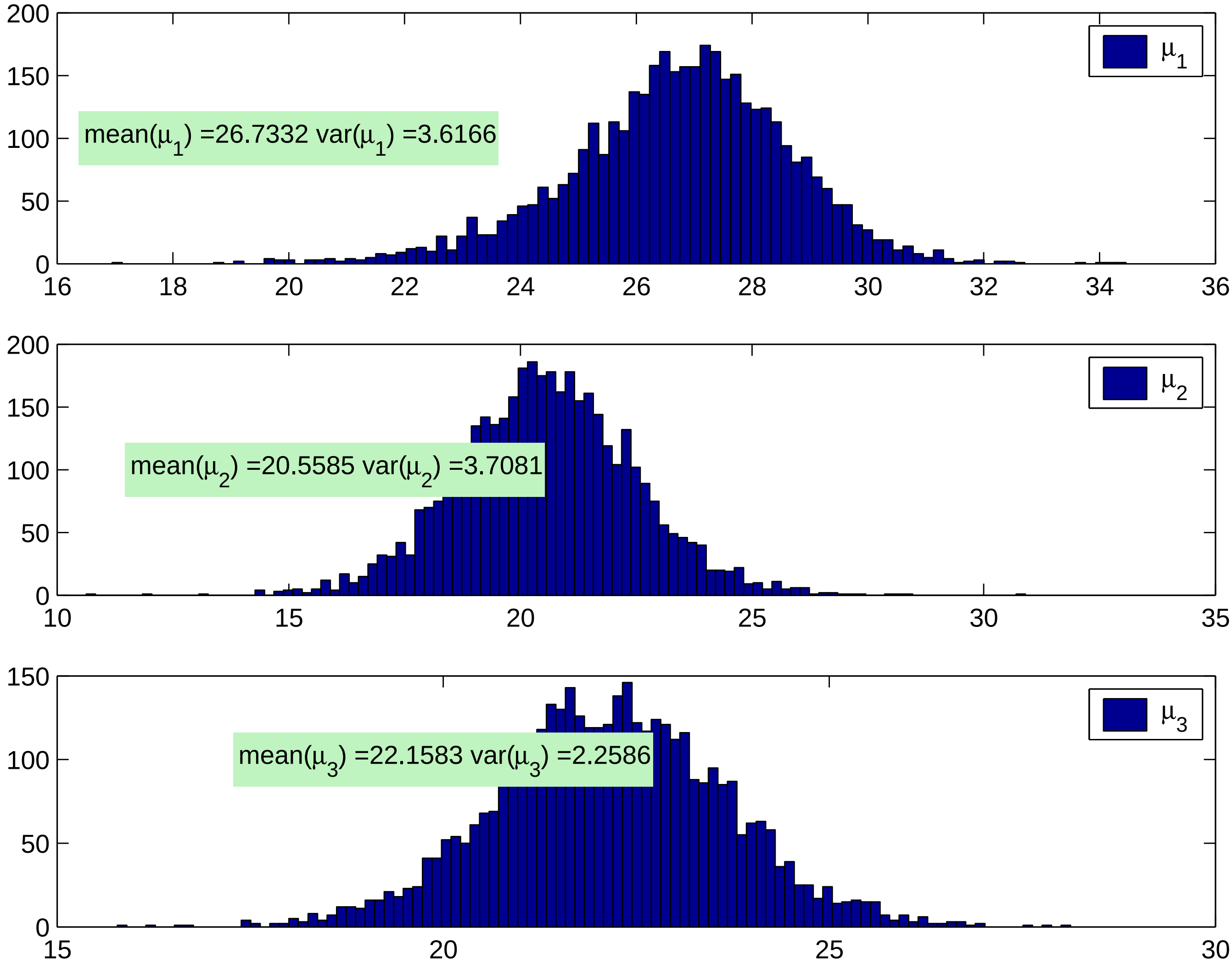

Fig. 1. Posterior Distribution of μ_1, μ_2 and μ_3

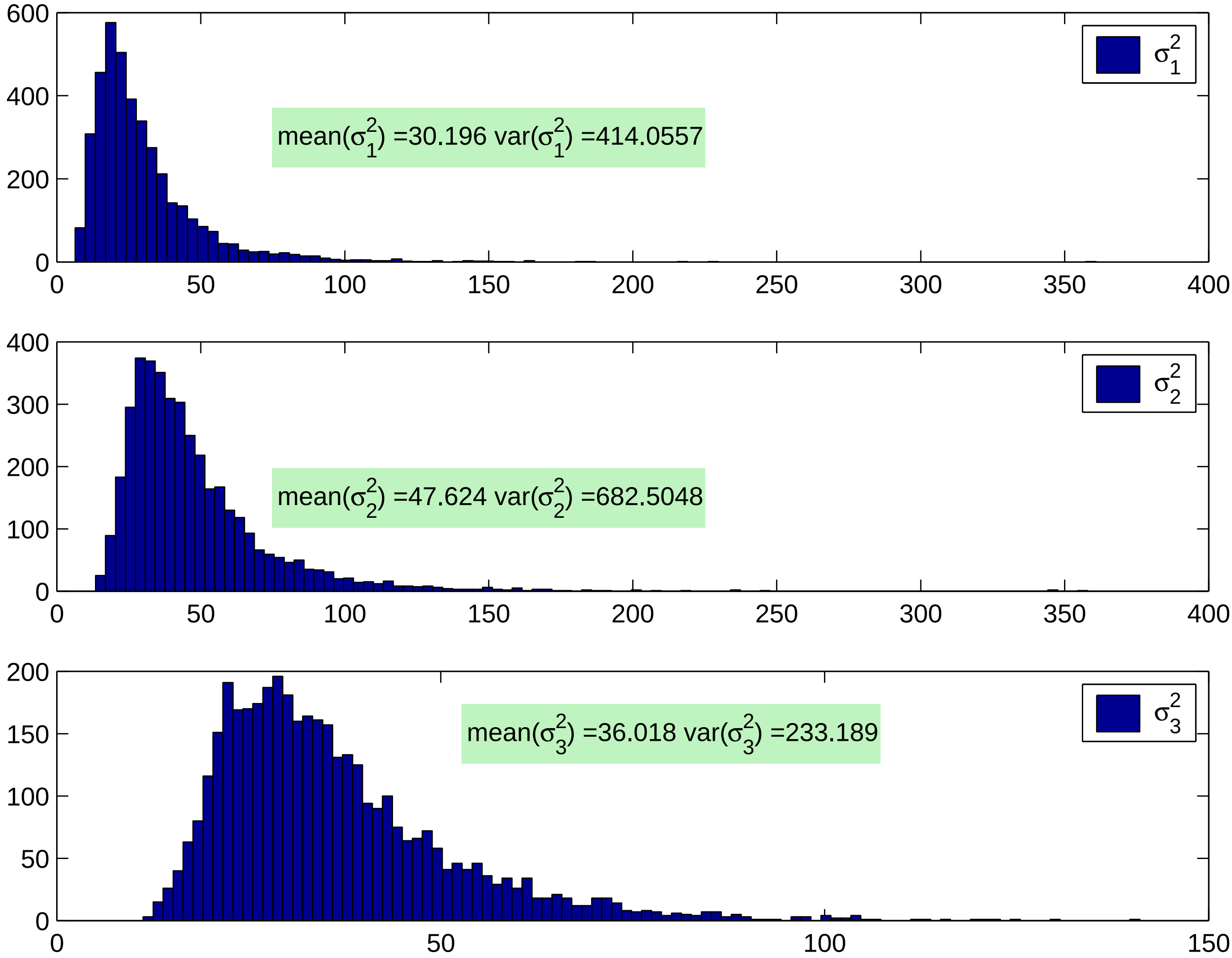

Fig. 2. Posterior Distribution of σ_1^2, σ_2^2 and σ_3^2

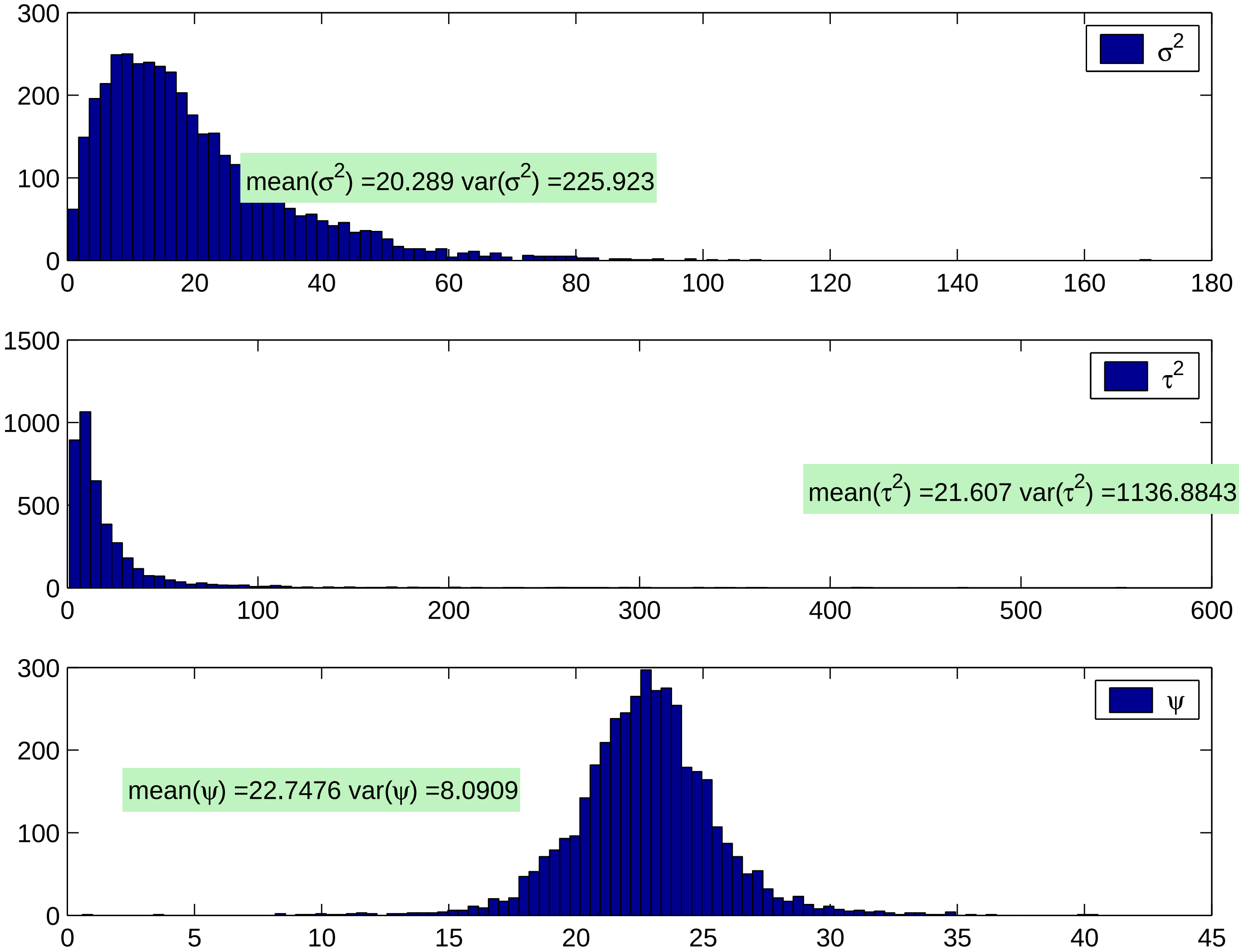

Fig. 3. Posterior Distribution of τ^2, σ^2 and ψ

2. SOLUTION TO PROBLEM 2

- the joint distribution of y, μ, σ is the following,

$$f(y,\mu,\sigma) \quad \propto \quad \prod_{k=1}^{K}(\frac{1}{\sigma}\exp{(-\frac{y_i-\mu}{\sigma})}\exp{(-\exp{(-\frac{y_i-\mu}{\sigma})}))} * \exp{(-\frac{\mu^2}{200})} * \frac{1}{\sigma}\exp{(-\frac{(log\sigma)^2}{200})}$$

so $\pi(\mu,\sigma|y_1,y_2,\cdots,y_n)$ is also proportional to the joint distribution.

$$\pi(\mu,\sigma|y_1,y_2,\cdots,y_n) \quad \propto \quad \prod_{k=1}^{K}(\frac{1}{\sigma}\exp{(-\frac{y_i-\mu}{\sigma})}\exp{(-\exp{(-\frac{y_i-\mu}{\sigma})}))} * \exp{(-\frac{\mu^2}{200})} * \frac{1}{\sigma}\exp{(-\frac{(log\sigma)^2}{200})}$$

- Here I use the two variables Normal distribution as the proposal distribution. Because the kernel is symmetric. So it's much easier to sampling from the two variable Normal distribution.

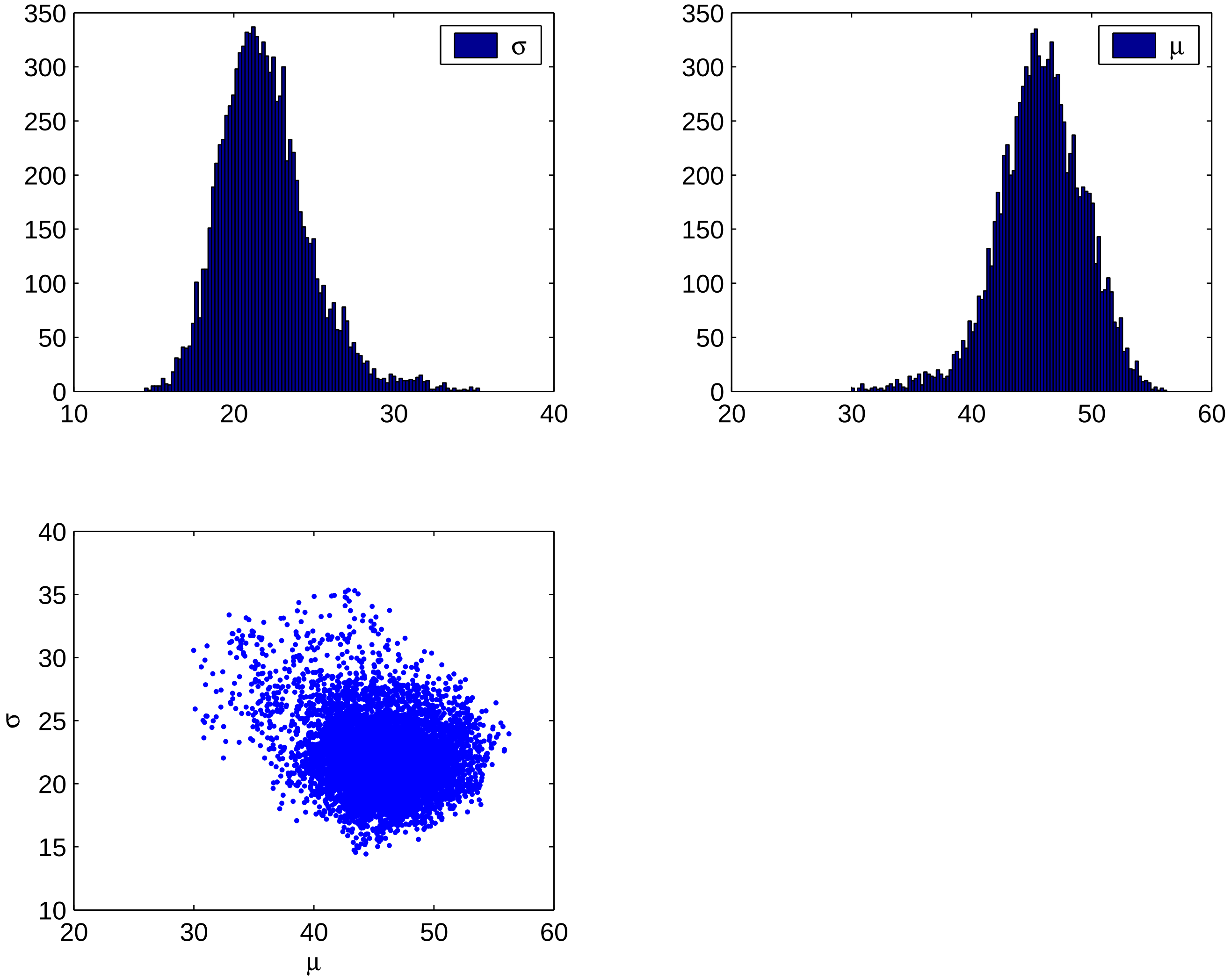

Fig. 4. Posterior Distribution of μ, σ and their scatterplot

```
function midterm2
clear all ;
```

```
close all ;

% set random generator seed
randn('seed',1)
sigmas=[];
mus=[];
sigma_old = 1 ;
mu_old    = 0 ;
Sigma=[1 0; 0 1];
M = 11000 ;
burn = 1000 ;

for ii = 1 : M
    prop = rand_MVN(1, [mu_old, sigma_old]', Sigma)'; %proposal from
    mu_prop    = prop(1);
    sigma_prop = prop(2);

    u = rand(1,1);
    post_p = post(mu_prop, sigma_prop) ;
    post_o = post(mu_old, sigma_old) ;

    mu_new = mu_old ;
    sigma_new = sigma_old ;

    if u <= min(post_p/post_o, 1)
        mu_new = mu_prop;
        sigma_new = sigma_prop;
        mu_old = mu_prop;
        sigma_old = sigma_prop;
    end
    mus = [mus mu_new];
    sigmas = [sigmas, sigma_new] ;
end

musdata = mus(burn+1 : M ) ;
sigmadata = sigmas(burn+1:M) ;

save mus.txt musdata -ascii;
save sigmas.txt sigmadata -ascii;

function eresult = e_result(mu, sigma)

y = [1951 154
1952 49.6
1953 46.7
1954 58.3
1955 70.5
1956 90
1957 70.1
1958 105.7
1959 37.4
1960 40.8
1961 34.7
```

```
1962 58.9
1963 72.2
1964 30
1965 71.6
1966 100
1967 33.7
1968 49.9
1969 56.1
1970 142.3
1971 28.6
1972 54.8
1973 74.1
1974 60
1975 50.9
1976 38.6
1977 53.4
1978 132.5
1979 50.7
1980 40.8
1981 84.3
1982 38.8
1983 27.4
1984 67
1985 118.7
1986 23.2
1987 55
1988 67.9
1989 87.3
1990 89
1991 98.7
1992 47.1
1993 71.6
1994 83.6
1995 44.3
1996 41.2
1997 35.9
1998 44.3
1999 410.4];

data = y(:,2) ;
for ii = 1 : 48
    eresult(ii) = exp((mu - data(ii))/sigma) ;
end

function result = post(mu, sigma)
eresult = e_result(mu, sigma) ;
result = 1 ;
temp = 0 ;
for ii = 1 : 48
    result = result * eresult(ii) / sigma ;
    temp   = temp + eresult(ii) ;
end
result = result * exp(-temp) ;
```

```
result = result * exp(-mu^2/200) ;
result = result * exp(-(log(sigma))^2/200) / sigma ;
```

- from the previous simulation, we can get the mean value of μ and σ from the corresponding posterior distribution.

$$
\begin{aligned}
\hat{\mu} &= 45.8 \\
\hat{\sigma} &= 22 \\
P(y^* >= 410|y) &\simeq 1 - F(410|\hat{\mu}, \hat{\sigma}) = 6.5 * E - 8
\end{aligned}
$$

3. SOLUTION TO PROBLEM 3

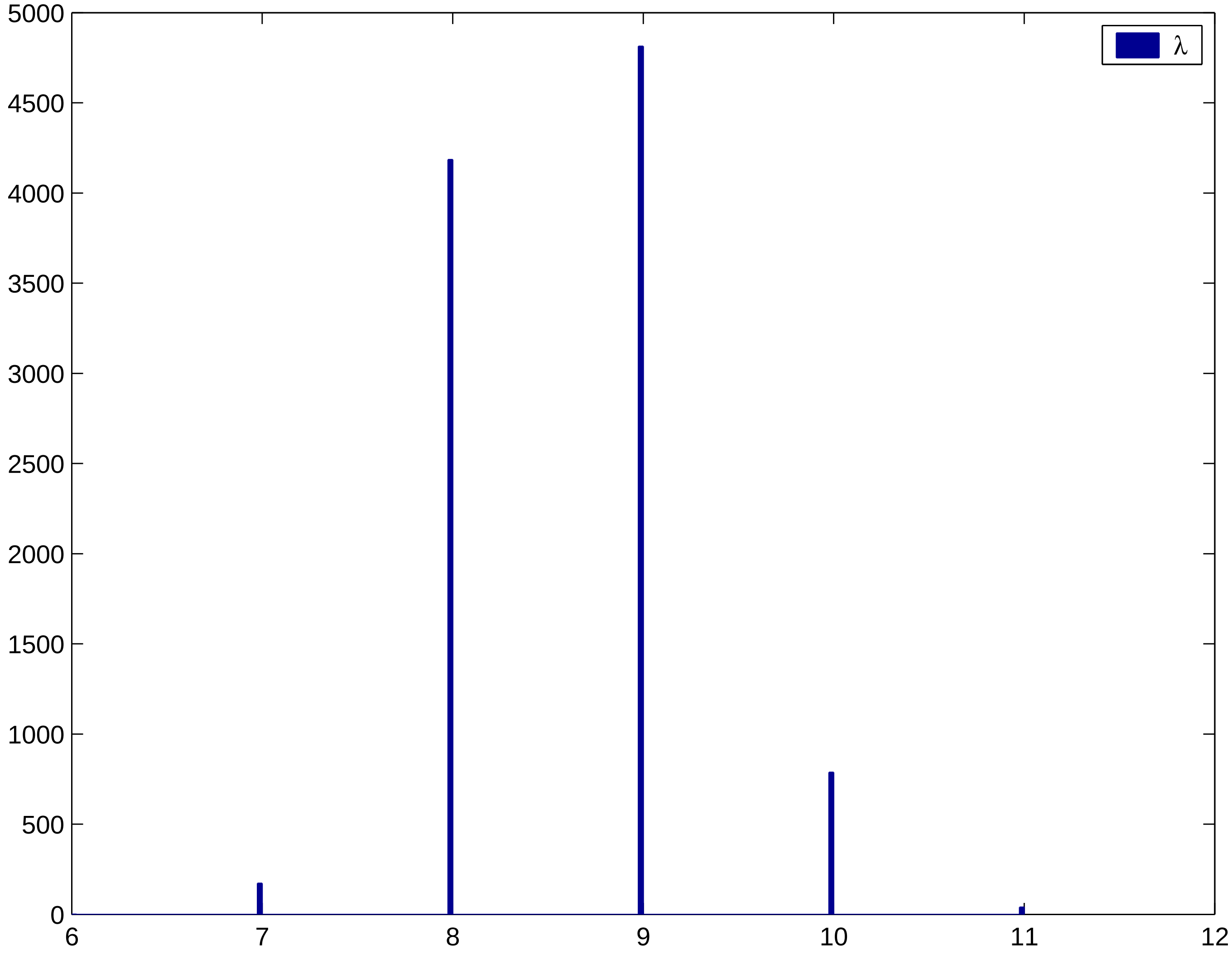

Fig. 5. Posterior distribution of λ

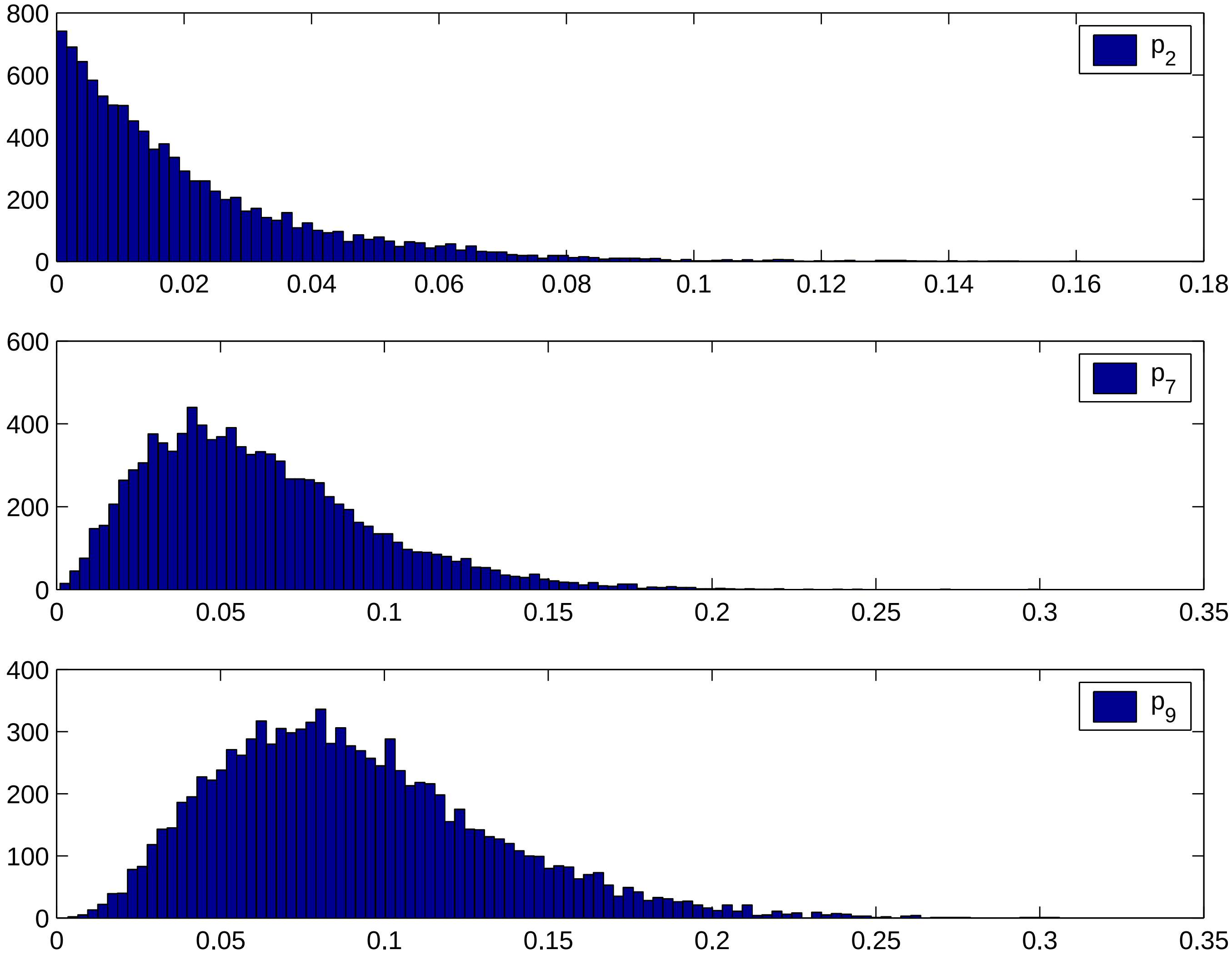

Fig. 6. Posterior distribution of $P[2], P[7], P[9]$

Table 1. Parameters Statistics

node	mean	sd	MC error	2.5%	median	97.5%
λ	8.634	0.6699	0.007177	8.0	9.0	10.0
P[2]	0.02044	0.02009	1.912E-4	5.233E-4	0.0142	0.07293
P[7]	0.06148	0.03415	3.454E-4	0.01251	0.0556	0.1435
P[9]	0.09073	0.04173	4.351E-4	0.02666	0.08504	0.187
P[10]	0.1038	0.04328	4.15E-4	0.03618	0.09834	0.2022
P[13]	0.08221	0.03893	4.355E-4	0.02328	0.07654	0.1731
P[17]	0.0203	0.02013	2.021E-4	5.283E-4	0.01424	0.07468

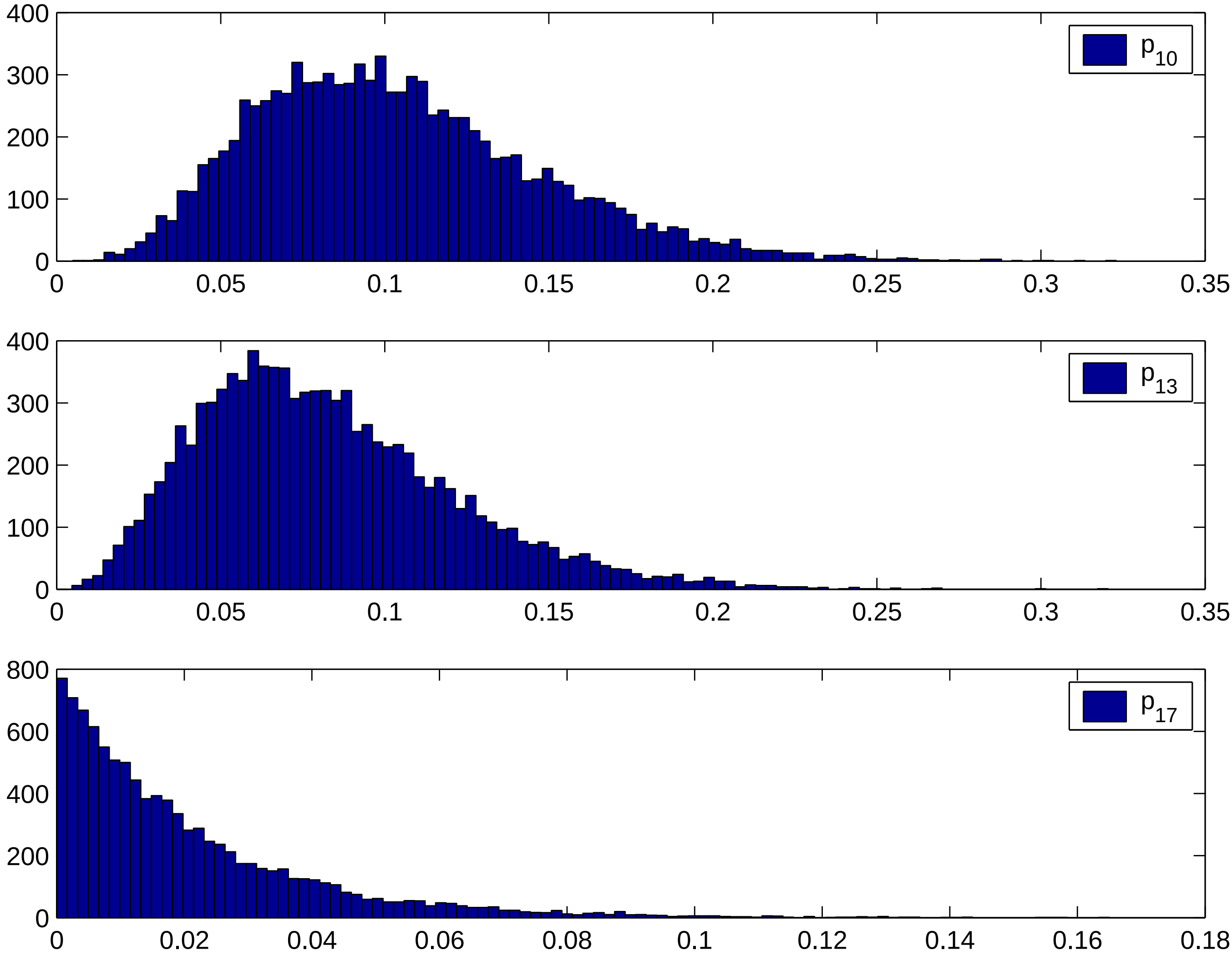

Fig. 7. Posterior distribution of $P[10], P[13], P[17]$