MIDTERM EXAM

ISyE6420

Released October 16, 12:00pm – due October 23, 11:55pm. This exam is not proctored and not time limited except the due date. Late submissions will not be accepted.

Use of all available electronic and printed resources is allowed except direct communication that violates Georgia Tech Academic Integrity Rules.

Name ________________________________

Problem	Emily and Car	Until 4th Success	Emperor Penguins	Total
Score	/33	/33	/34	/100

1. Emily, Car, Stock Market, Sweepstakes, Vacation and Bayes. Emily is taking Bayesian Analysis course. She believes she will get an A with probability 0.6, a B with probability 0.3, and a C or less with probability 0.1. At the end of semester she will get a car as a present form her (very) rich uncle depending on her class performance. For getting an A in the course Emily will get a car with probability 0.8, for B with probability 0.5, and for anything less than B, she will get a car with probability of 0.2. These are the probabilities if the market is bullish. If the market is bearish, the uncle is less likely to make expensive presents, and the above probabilities are 0.5, 0.3, and 0.1, respectively. The probabilities of bullish and bearish market are equal, 0.5 each. If Emily gets a car, she would travel to Redington Shores with probability 0.7, or stay on campus with probability 0.3. If she does not get a car, these two probabilities are 0.2 and 0.8, respectively. Independently, Emily may be a lucky winner of a sweepstake lottery for a free air ticket and vacation in hotel Sol at Redington Shores. The chance to win the sweepstake is 0.001, but if Emily wins, she will go to vacation with probability of 0.99, irrespective of what happened with the car.

Figure 1: Emily on the road

After the semester was over you learned that Emily is at Redington Shores.
(a) What is the probability that she got a car?
(b) What is the probability that she won the sweepstakes?
(c) What is the probability that she got a B in the course?
(d) What is the probability that the market was bearish?

Hint: You can solve this problem by any of the 3 ways: (i) use of WinBUGS or OpenBUGS, (ii) direct simulation using Octave/MATLAB, R, or Python, and (iii) exact calculation. Use just one of the three ways to solve it. WinBUGS/OpenBUGS or direct simulation are recommended. The exact solution, although straightforward, may be quite messy.

2. Trials until Fourth Success. The number of failures until the fourth success in a series of independent trials is observed in 11 independent experiments: [1] 5, 2, 2, 0, 1, 4, 3,

[1] You may link this to your favorite story, like, on multiple occasions Larry Bird shoots from a distance until he makes the 4th score, number of misses is counted

5, 0, 7, and 1.

You are interested in probability p of a success in a single trial. Assume that p is given a beta prior with parameters

(a) $a = b = 1$;

(b) $a = b = 1/2$;

(c) $a = 9,\ b = 1$.

For each case (a)-(c) find:

Bayes estimator of p, the 95% credible set for p, and the posterior probability of hypothesis $H : p \geq 0.8$.

Hint: No WinBUGS should be used, the problem is conjugate. You will need Octave or R or Python, to calculate beta cdf and quantiles.

3. Penguins. A researcher is interested in testing whether the mean height of Emperor penguins (*Aptenodytes forsteri*) from a small island is less than $\mu = 45$ in., which is believed to be the average height for the whole Emperor penguin population.

Figure 2: Emperor Penguins

The heights were measured of 14 randomly selected adult birds from the island with the following results:

41	44	43	47	43	46	45	42	45	45	43	45	47	40

Assume that the measurements are normal $\mathcal{N}(\mu, 1/\tau)$ where the parameter μ is given normal prior $\mathcal{N}(45, 2^2)$ and the precision parameter τ is given gamma $\mathcal{G}a(4, 1/2)$ prior. The gamma distribution is parameterized by the rate parameter, in this case equal to 1/2.

Develop Gibbs Sampler that will sample from the posteriors for μ and τ.

Burn in the fist 1000 simulations and simulate additional 10,000 μs and τs. From the simulated values

(a) approximate the posterior probability of hypothesis that the researcher was interested in, $H_0 : \mu < 45$;

(b) approximate the 95% equitailed credible set for parameter τ.

Sol. (a) 0.9805; (b) [0.1717, 0.6023]